2) If $a+x=b+y=c+z+1$, where a,b,c,x,y,z are non-zero distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to:

(a) $y(b-a)$ (b) $y(a-b)$ (c) 0 (d) $y(a-c)$ [Main Sep. 05, 2020 (II)]

Solution. (b)

Using properties of determinant

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} + y\begin{vmatrix} x & 1 & x+a \\ y & 1 & y+b \\ z & 1 & z+c \end{vmatrix}$$

$$= 0 + y\begin{vmatrix} x & 1 & x+a \\ y-x & 0 & 0 \\ z-x & 0 & -1 \end{vmatrix} \quad \begin{bmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{bmatrix}$$

$$= -y(x-y) = -y(b-a) = y(a-b)$$