

Let the numbers $2, b, c$ be in A.P. and

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If $\det(A) \in [2, 16]$, then c lies in the interval:

(a) $[2, 3)$

(b) $(2 + 2^{2/4}, 4)$

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(c) $[4, 6)$

(d) $[3, 2 + 2^{3/4}]$

Solution:

(c) Consider, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$

$$C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$$

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & (b-2)(b+2) & (c-2)(c+2) \end{vmatrix}$$

$$= (b-2)(c-2)(c-b)$$

$\because 2, b, c$ are in A.P.

$$\therefore (b-2) = (c-b) = d \text{ and } c-2 = 2d$$

$$\Rightarrow |A| = d \cdot 2d \cdot d = 2d^3$$

$$\therefore |A| \in [2, 16] \Rightarrow 1 \leq d^3 \leq 8 \Rightarrow 1 \leq d \leq 2$$

$$4 \leq 2d + 2 \leq 6 \Rightarrow 4 \leq c \leq 6$$