Concepts and Formulas to Remember :

Determinant

Every square matrix A is associated with a number, called its determinant and it is denoted by det (A) or |A| .

Only square matrices have determinants. The matrices which are not square do not have determinants

(i) **First Order Determinant**

If A = [a], then det (A) = |A| = a

(ii) **Second Order Determinant**

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},$$

$|A| = a_{11}a_{22} - a_{21}a_{12}$

(iii) **Third Order Determinant**

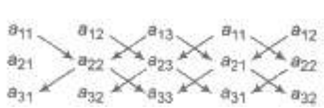
$$\text{If } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \text{ then}$$

$$|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\text{or } |A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Evaluation of Determinant of Square Matrix of Order 3 by Sarrus Rule

$$\text{If } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$ then determinant can be formed by enlarging the matrix by adjoining the first two columns on the right and draw lines as show below parallel and perpendicular to the diagonal.



The value of the determinant, thus will be the sum of the product of element. in line parallel to the diagonal minus the sum of the product of elements in line perpendicular to the line segment. Thus,

$\Delta = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$.

Note This method doesn’t work for determinants of order greater than 3.

Properties of Determinants

(i) The value of the determinant remains unchanged, if rows are changed into columns and columns are changed into rows e.g.,

$|A'| = |A|$

(ii) If $A = [a_{ij}]_{n \times n}$, $n > 1$ and B be the matrix obtained from A by interchanging two of its rows or columns, then

$\det(B) = -\det(A)$

(iii) If two rows (or columns) of a square matrix A are proportional, then $|A| = O$.

(iv) $|B| = k\,|A|$,where B is the matrix obtained from A, by multiplying one row (or column) of A by k.

(v) $|kA| = k^n|A|$, where A is a matrix of order n x n.

(vi) If each element of a row (or column) of a determinant is the sum of two or more terms, then the determinant can be expressed as the sum of two or more determinants, e.g.,

$$\begin{vmatrix} a_1 + a_2 & b & c \\ p_1 + p_2 & q & r \\ u_1 + u_2 & v & \end{vmatrix} = \begin{vmatrix} a_1 & b & c \\ p_1 & q & r \\ u_1 & v & \end{vmatrix} + \begin{vmatrix} a_2 & b & c \\ p_2 & q & r \\ u_2 & v & \end{vmatrix}$$

(vii) If the same multiple of the elements of any row (or column) of a determinant are added to the corresponding elements of any other row (or column), then the value of the new determinant remains unchanged, e.g.,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} + ka_{31} & a_{12} + ka_{32} & a_{13} + ka_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

(viii) If each element of a row (or column) of a determinant is zero, then its value is zero.

(ix) If any two rows (columns) of a determinant are identical, then its value is zero.