Previous Year Question with Solution:

If a + x = b + y = c + z + 1, where a, b, c, x, y, z are non-zero distinct real numbers, then

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$$

is equal to:

- (1) 0
- (2) y(a-b)
- (3) y(b-a)
- (4) y(a-c)

Soln:

$$a + x = b + y = c + z + 1$$

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} \qquad C_3 \rightarrow C_3 - C_3$$

$$\begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix} \qquad C_2 \rightarrow C_2 - C_3$$

$$\begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} \qquad R_3 \rightarrow R_3 - R_1, R_2 \rightarrow R_2 - R_1$$

$$\begin{vmatrix} x & y & a \\ y - x & 0 & b - a \\ z - x & 0 & c - a \end{vmatrix}$$

$$= (-y)[(y - x) (c - a) - (b - a) (z - x)]$$

$$= (-y)[(a - b) (c - a) + (a - b) (a - c - 1)]$$

$$= (-y)[(a - b) (c - a) + (a - b) (a - c) + b - a)$$

$$= -y(b - a) = y(a - b)$$