Previous Year Question with Solution :

If $a_r = \cos\frac{2r\pi}{9} + i\,sin\frac{2r\pi}{9}$

$a_r = \cos 2r\pi/9 + i \sin 2r\pi/9$, $r = 1, 2, 3, \ldots$, $i = \sqrt{-1}$ then the determinant

$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to :

(1) $a_2a_6 - a_4a_8$

(2) a_9

(3) $a_1a_9 - a_3a_7$

(4) a_5

Soln :

$$a_r = e^{\frac{i2\pi r}{9}},$$

$r = 1, 2, 3, \ldots$ $a_1, a_2, a_3, \ldots$

. are in G.P.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_n & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} a_1 & a_2^2 & a_1^3 \\ a_1^4 & a_1^5 & a_1^6 \\ a_1^7 & a_1^8 & a_1^9 \end{vmatrix}$$

$$= a_1 . a_1^4 . a_1^7 \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \end{vmatrix} = 0$$

$$\text{Now } a_1a_9 - a_3a_7 = a_1^{10} - a_1^{10} = 0$$