Previous Year Question with Solution :

Let A = $\begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, a ∈ R be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to :

(1) 36

(2) 24

(3) 45

(4) 18

Soln :

$A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}, a \in R$

and $P = \dfrac{A + A^T}{2} = \begin{bmatrix} 2 & \dfrac{3+a}{2} \\ \dfrac{a+3}{2} & 0 \end{bmatrix}$

and $Q = \dfrac{A - A^T}{2} = \begin{bmatrix} 0 & \dfrac{3-a}{2} \\ \dfrac{a-3}{2} & 0 \end{bmatrix}$

As, det (Q) = 9

$\Rightarrow (a-3)^2 = 36$

$\Rightarrow a = 3 \pm 6$

$\therefore \boxed{a = 9, -3}$

$\therefore \quad \det.(P) = \begin{vmatrix} 2 & \dfrac{3+a}{2} \\ \dfrac{a+3}{2} & 0 \end{vmatrix}$

$= 0 - \dfrac{(a-3)^2}{4} = 0,$ for $a = -3$

$= 0 - \dfrac{(a-3)^2}{4} = -\dfrac{1}{4}(12)(12),$ for $a = 9$

∴ Modulus of the sum of all possible values of det. (P) = |-36|+|0| = 36