Previous Year Question with Solution :

Let A = [a_{ij}] be a 3 × 3 matrix, where

$$a_{ij} = \begin{cases} 1, & if\ i = j \\ -x, & if\ |i-j| = 1 \\ 2x+1, & otherwise. \end{cases}$$

1, if i = j

-x, if |i - j| = 1

2x+1, otherwise.

Let a function f : R → R be defined as f(x) = det(A). Then the sum of maximum and minimum values of f on R is equal to:

(1) -20/27

(2) 88/27

(3) 20/27

(4) -88/27

Soln :

$$A = \begin{bmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{bmatrix}$$

$$|A| = 4x^3 - 4x^2 - 4x = f(x)$$

$$f'(x) = 4(3x^2 - 2x - 1) = 0$$

$$\Rightarrow x = 1\ ;\ x = \frac{-1}{3}$$

$$\therefore\ \underbrace{f(1) = -4}_{\min}\ ;\underbrace{f\left(-\frac{1}{3}\right) = \frac{20}{27}}_{\max.}$$

$$\text{Sum} = -4 + \frac{20}{27} = -\frac{88}{27}$$