4) Solve the following linear equation by rank method

$x + 9y - z = 27 \qquad x - 8y + 16z = 10 \qquad 2x + y + 15z = 37$

Solution:
$$A = \begin{bmatrix} 1 & 9 & -1 \\ 1 & -8 & 16 \\ 2 & 1 & 15 \end{bmatrix} \quad B = \begin{bmatrix} 27 \\ 10 \\ 37 \end{bmatrix}$$

$$[A, B] \sim \left[\begin{array}{ccc|c} 1 & 9 & -1 & 27 \\ 1 & -8 & 16 & 10 \\ 2 & 1 & 15 & 37 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 9 & -1 & 27 \\ 0 & -17 & 17 & -17 \\ 0 & -17 & 17 & -17 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 9 & -1 & 27 \\ 0 & -17 & 17 & -17 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\text{Rank}(A) = \text{Rank}([A|B]) = 2 < 3$

The system is consistent and it has infinitely many solution.

$x + 9y - z = 27$ — ①

$17y + 17z = -17$ — ②

$\Rightarrow \cancel{x + y = t} \Rightarrow y + z = -1.$

Put $z = t$

$y = -1 - t.$

Putting the value of y and z in ①,

$x + 9(-1-t) - t = 27$

$x = 27 + 9 - 8t$

$x = 36 - 8t$

$x = 36 - 8t,\ y = -1 - t$ and $z = t$ where $t \in \mathbb{R}$