

3) solve the following equation using the rank method.

$$4x - 2y + 5z = 6$$

$$3x + 3y + 8z = 4, \quad x - 5y - 3z = 5$$

solution: $A = \begin{bmatrix} 4 & -2 & 5 \\ 3 & 3 & 8 \\ 1 & -5 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 4 \\ 5 \end{bmatrix}$

$$[A|B] \sim \left[\begin{array}{ccc|c} 4 & -2 & 5 & 6 \\ 3 & 3 & 8 & 4 \\ 1 & -5 & -3 & 5 \end{array} \right]$$

$$R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & -5 & -3 & 5 \\ 3 & 3 & 8 & 4 \\ 4 & -2 & 5 & 6 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 3R_1$$

$$R_3 \rightarrow R_3 - 4R_1$$

$$\left[\begin{array}{ccc|c} 1 & -5 & -3 & 5 \\ 0 & 18 & 17 & -11 \\ 0 & 18 & 17 & -14 \end{array} \right]$$

$$R_3 \rightarrow R_3 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & -5 & -3 & 5 \\ 0 & 18 & 17 & -11 \\ 0 & 0 & 0 & -3 \end{array} \right]$$

$$\text{Rank}(A) = 2$$

$$\text{Rank}([A|B]) = 3,$$

The system is inconsistent and it has no solution.