Question: -

In a ΔABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\sin^2 \frac{A}{2}$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C respectively, then **(2009)**

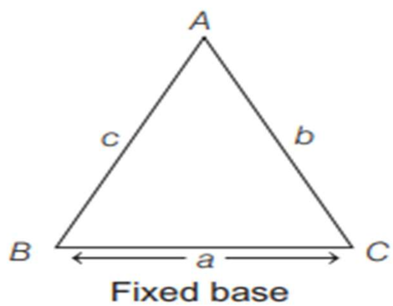
(a) $b + c = 4a$ (b) $b + c = 2a$

(c) locus of point A is an ellipse

(d) locus of point A is a pair of straight line

Solution: -

Given, $\cos B + \cos C = 4\sin^2 \frac{A}{2}$



$\Rightarrow\ 2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right) = 4\sin^2\frac{A}{2}$

$\Rightarrow 2\sin\frac{A}{2}\left[\cos\left(\frac{B-C}{2}\right) - 2\sin\frac{A}{2}\right] = 0$

$\Rightarrow\ \cos\left(\frac{B-C}{2}\right) - 2\cos\left(\frac{B+C}{2}\right) = 0$ as $\sin\frac{A}{2} \neq 0$

$\Rightarrow\ -\cos\frac{B}{2}\cos\frac{C}{2} + 3\sin\frac{B}{2}\sin\frac{C}{2} = 0$

$\Rightarrow\ \tan\frac{B}{2}\tan\frac{C}{2} = \frac{1}{3}$

$\Rightarrow\ \sqrt{\frac{(s-a)(s-c)}{s(s-b)} \cdot \frac{(s-b)(s-a)}{s(s-c)}} = \frac{1}{3}$

$\Rightarrow\ \frac{s-a}{s} = \frac{1}{3} \Rightarrow 2s = 3a$

$\Rightarrow\ b + c = 2a$

$\therefore$ Locus of A is an ellipse.