Question: -

If in a $\triangle ABC$,

$$\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$$

Then, the value of the $\angle A$ is ...... degree. **(1993, 2M)**

Solution: -

Given, $\dfrac{2\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{2\cos C}{c} = \dfrac{a}{bc} + \dfrac{b}{ca}$ ...(i)

We know that, $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ac}$$

and $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

On putting these values in Eq. (i), we get

$$\frac{2\,(b^2 + c^2 - a^2)}{2abc} + \frac{c^2 + a^2 - b^2}{2abc}$$

$$+ \frac{2(a^2 + b^2 - c^2)}{2abc} = \frac{a}{bc} + \frac{b}{ca}$$

$$\Rightarrow \quad \frac{2\,(b^2 + c^2 - a^2) + c^2 + a^2 - b^2 + 2\,(a^2 + b^2 - c^2)}{2abc}$$

$$= \frac{a^2 + b^2}{abc}$$

$$\Rightarrow \quad 3b^2 + c^2 + a^2 = 2a^2 + 2b^2$$

$$\Rightarrow \quad b^2 + c^2 = a^2$$

Hence, the angle A is 90°.