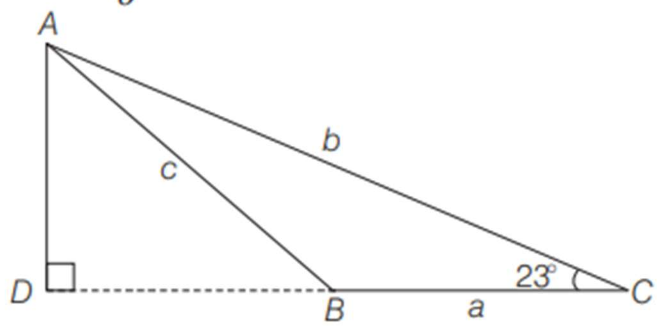
Question: -

In a ΔABC, AD is the altitude from A. Given $b > c, \angle C = 23°$ and $AD = \dfrac{abc}{b^2 - c^2}$, then $\angle B = \ldots$.

(1994, 2M)

Solution: -

In $\Delta ADC, \dfrac{AD}{b} = \sin 23°$



$$\Rightarrow \qquad AD = b \sin 23°$$

But $$AD = \frac{abc}{b^2 - c^2} \qquad \text{[given]}$$

$$\Rightarrow \qquad \frac{abc}{b^2 - c^2} = b \sin 23°$$

$$\Rightarrow \qquad \frac{a}{b^2 - c^2} = \frac{\sin 23°}{c} \qquad \ldots\text{(i)}$$

Again, in ΔABC,

$$\frac{\sin A}{a} = \frac{\sin 23°}{c}$$

$$\Rightarrow \qquad \frac{\sin A}{a} = \frac{a}{b^2 - c^2} \qquad \text{[from Eq. (i)]}$$

$$\Rightarrow \qquad \sin A = \frac{a^2}{b^2 - c^2}$$

$$\Rightarrow \sin A = \frac{k^2 \sin^2 A}{k^2 \sin^2 B - k^2 \sin^2 C}$$

$$\Rightarrow \qquad \sin A = \frac{\sin^2 A}{\sin^2 B - \sin^2 C}$$

$$\Rightarrow \qquad \sin A = \frac{\sin^2 A}{\sin (B + C) \sin (B - C)}$$

$$\Rightarrow \qquad \sin A = \frac{\sin^2 A}{\sin A \cdot \sin (B - C)}$$

$$\Rightarrow \qquad \sin (B - C) = 1 \qquad [\because \sin A \neq 0]$$

$$\Rightarrow \qquad \sin (B - 23^\circ) = \sin 90^\circ$$

$$\Rightarrow \qquad B - 23^\circ = 90^\circ$$

$$\therefore \qquad B = 113^\circ$$