

Question: -

If in a triangle ABC , $a = 1 + \sqrt{3}$ cm, $b = 2$ cm and $\angle C = 60^\circ$, then find the other two angles and the third side.
(1978, 3M)

Solution: -

Given that,

$$a = 1 + \sqrt{3}, b = 2 \text{ and } \angle C = 60^\circ$$

$$\text{We have, } c^2 = a^2 + b^2 - 2ab \cos C$$

$$\Rightarrow c^2 = (1 + \sqrt{3})^2 + 4 - 2(1 + \sqrt{3}) \cdot 2 \cos 60^\circ$$

$$\Rightarrow c^2 = 1 + 2\sqrt{3} + 3 + 4 - 2 - 2\sqrt{3}$$

$$\Rightarrow c^2 = 6$$

$$\Rightarrow c = \sqrt{6}$$

Using sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{1 + \sqrt{3}}{\sin A} = \frac{2}{\sin B} = \frac{\sqrt{6}}{\sin 60^\circ}$$

$$\therefore \sin B = \frac{2 \sin 60^\circ}{\sqrt{6}} = \frac{2 \times \frac{\sqrt{3}}{2}}{\sqrt{6}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow B = 45^\circ$$

$$\therefore A = 180^\circ - (60^\circ + 45^\circ) = 75^\circ$$