Question: -

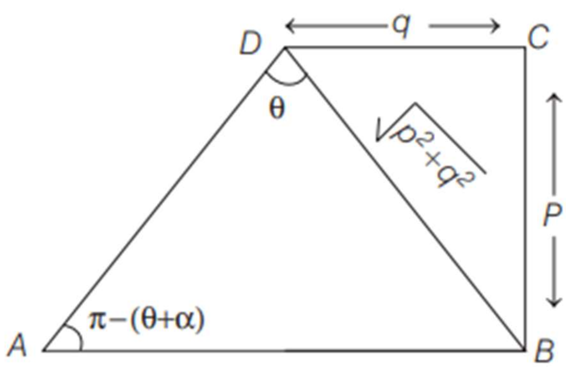
$ABCD$ is a trapezium such that AB and CD are parallel and $BC \perp CD$, if $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to **(2013 Main)**

(a) $\dfrac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (b) $\dfrac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$

(c) $\dfrac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$ (d) $\dfrac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

Solution: -

Applying sine rule in ΔABD,



$$\frac{AB}{\sin\theta} = \frac{\sqrt{p^2+q^2}}{\sin\{\pi-(\theta+\alpha)\}}$$

$$\Rightarrow \frac{AB}{\sin\theta} = \frac{\sqrt{p^2+q^2}}{\sin(\theta+\alpha)}$$

$$\Rightarrow AB = \frac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cos\alpha + \cos\theta\sin\alpha} \quad \left[\because \cos\alpha = \frac{q}{\sqrt{p^2+q^2}}\right]$$

$$= \frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$$

and $\sin\alpha = \dfrac{p}{\sqrt{p^2+q^2}}$