Question: -

If the angles of a triangle are in the ratio $4:1:1$, then the ratio of the longest side to the perimeter is

(2003, 1M)

(a) $\sqrt{3}:(2+\sqrt{3})$ (b) $1:3:2$

(c) $1:2+\sqrt{3}$ (d) $2:3$

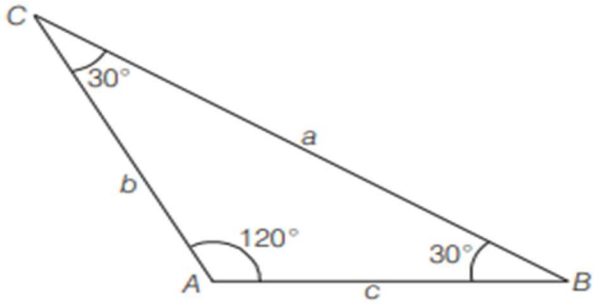
Solution: -

Given, ratio of angles are $4:1:1$.

$$\Rightarrow \quad 4x + x + x = 180^\circ$$

$$\Rightarrow \quad x = 30^\circ$$

$$\therefore \quad \angle A = 120^\circ, \quad \angle B = \angle C = 30^\circ$$



Thus, ratio of longest side to perimeter $= \dfrac{a}{a+b+c}$

Let $b = c = x$

$\Rightarrow \quad a^2 = b^2 + c^2 - 2bc\cos A$ [by cosine rule]

$\Rightarrow \quad a^2 = 2x^2 - 2x^2\cos A$

$= 2x^2(1 - \cos A)$

$\Rightarrow \quad a^2 = 4x^2\sin^2 A/2$

$\Rightarrow \quad a = 2x\sin A/2$

$\Rightarrow \quad a = 2x\sin 60^\circ = \sqrt{3}x$

Thus, required ratio

$$= \frac{a}{a+b+c}$$

$$= \frac{\sqrt{3}x}{x + x + \sqrt{3}x}$$

$$= \frac{\sqrt{3}}{2+\sqrt{3}}$$

$$= \sqrt{3}:2+\sqrt{3}$$