Question: -

The angles A, B and C of a ΔABC are in AP and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq cm) of this triangle is **(2019 Main, 10 April II)**

(a) $\frac{2}{\sqrt{3}}$ (b) $4\sqrt{3}$ (c) $2\sqrt{3}$ (d) $\frac{4}{\sqrt{3}}$

Solution: -

It is given that angles of a ΔABC are in AP.

So, $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle B - d + \angle B + \angle B + d = 180^\circ$

[if $\angle A$, $\angle B$ and $\angle C$ are in AP, then it taken as $\angle B - d$, $\angle B$, $\angle B + d$ respectively, where d is common difference of AP]

$\Rightarrow \quad 3\angle B = 180^\circ \quad \Rightarrow \quad \angle B = 60^\circ \quad \ldots\text{(i)}$

and $\quad \frac{a}{b} = \frac{1}{\sqrt{3}} \quad$ [given]

$\Rightarrow \quad \frac{\sin A}{\sin B} = \frac{1}{\sqrt{3}}$

$\left[\text{by sine rule } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}\right]$

$\Rightarrow \quad \frac{\sin A}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \quad \left[\because \sin B = \sin 60^\circ = \frac{\sqrt{3}}{2}\right]$

$\Rightarrow \quad \sin A = \frac{1}{2} \Rightarrow \angle A = 30^\circ$

So, $\quad \angle C = 90^\circ$

$\therefore$ From sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$\Rightarrow \quad \frac{a}{\frac{1}{2}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{4}{1} \quad [\because c = 4 \text{ cm}]$

$\Rightarrow \quad a = 2 \text{ cm}, b = 2\sqrt{3} \text{ cm}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} ab \sin C = \frac{1}{2} \times 2 \times 2\sqrt{3} \times 1$

$= 2\sqrt{3}$ sq. cm