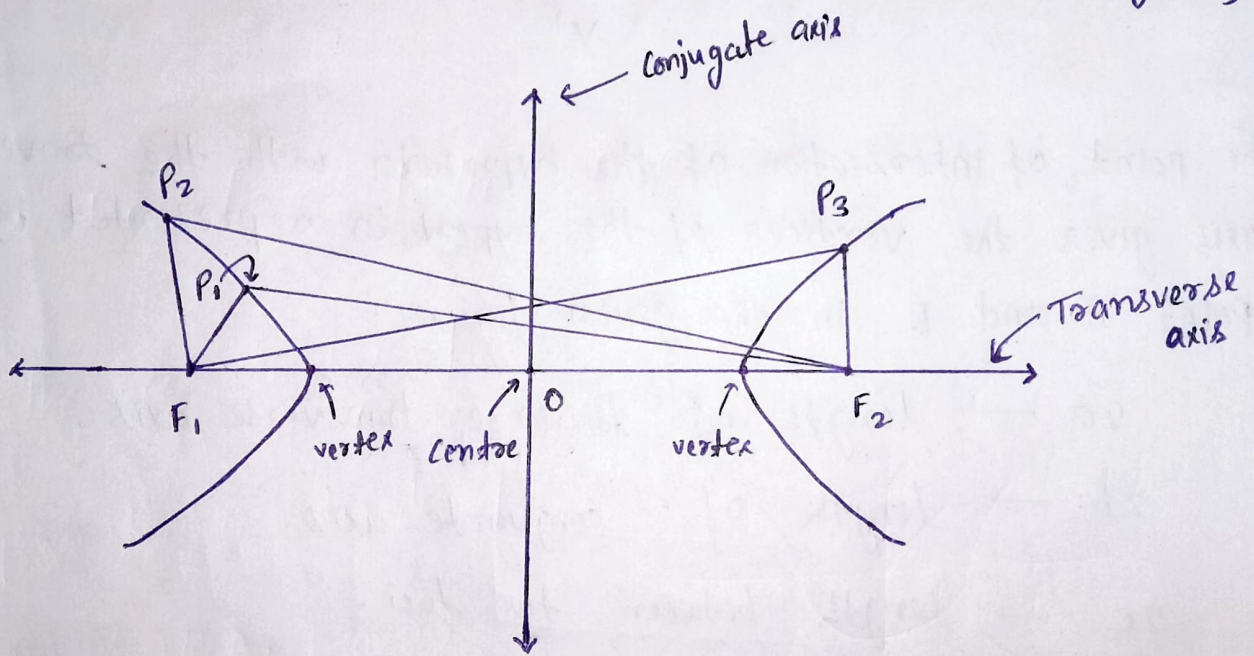


Notes - Hyperbola

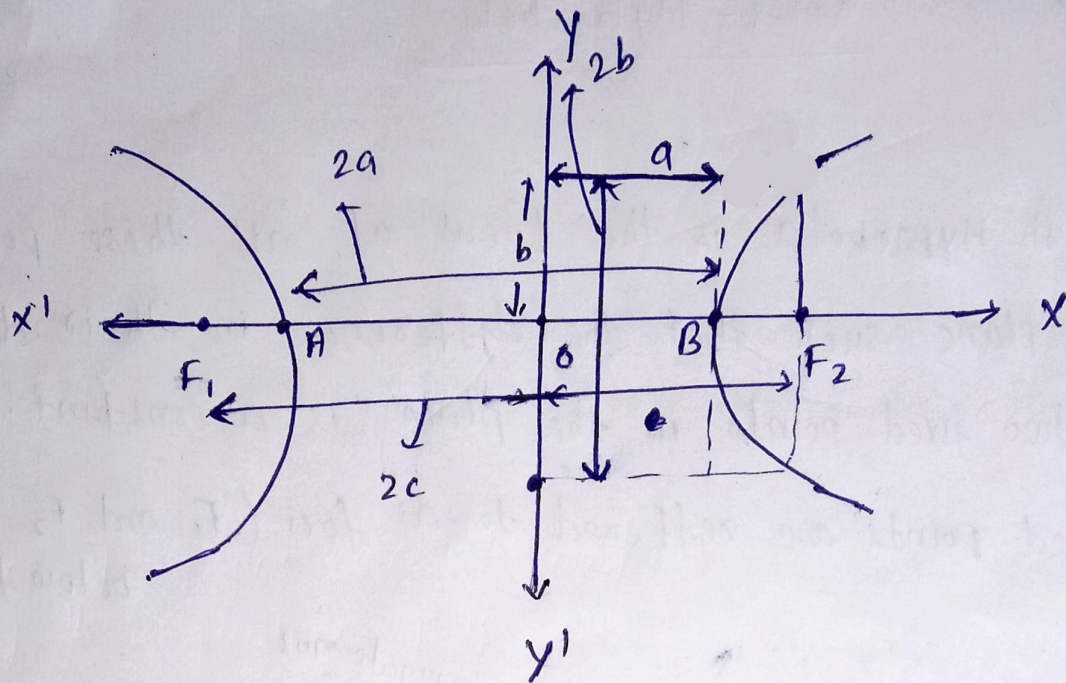
Defn :- A Hyperbola is the locus of all those points in a plane such that the difference in their distances from two fixed points in the plane is a constant.

The fixed points are referred to as foci (F_1 and F_2 in the below figure)



The above figure represents a hyperbola such that

$$P_1F_2 - P_1F_1 = P_2F_2 - P_2F_1 = P_3F_1 - P_3F_2 \text{ is constant}$$



The point of intersection of the hyperbola with the transverse axis gives the vertices of the hyperbola represented by the point A and B in the given figure.

$2a \rightarrow$ length of transverse axis.

$2b \rightarrow$ length of conjugate axis.

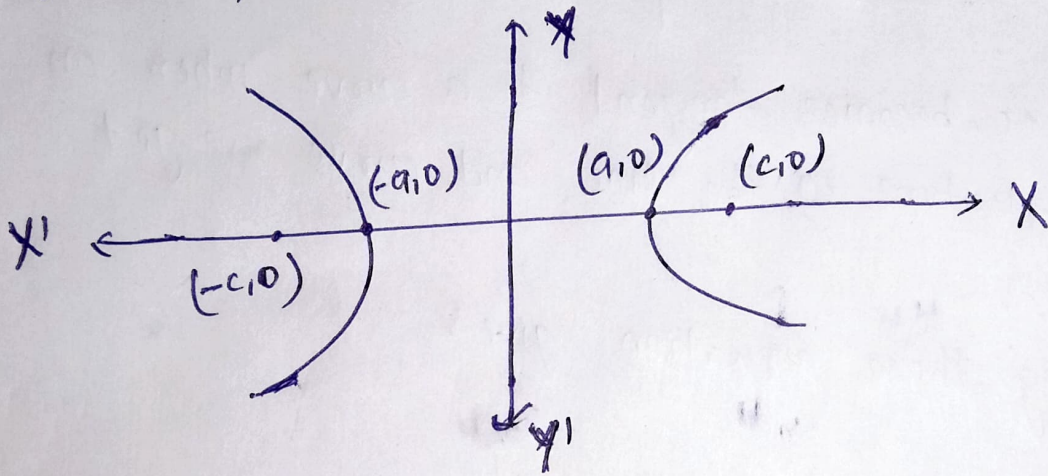
$2c \rightarrow$ length between two foci.

Relationship between a , b and c is given by

$$b = \sqrt{c^2 - a^2}$$

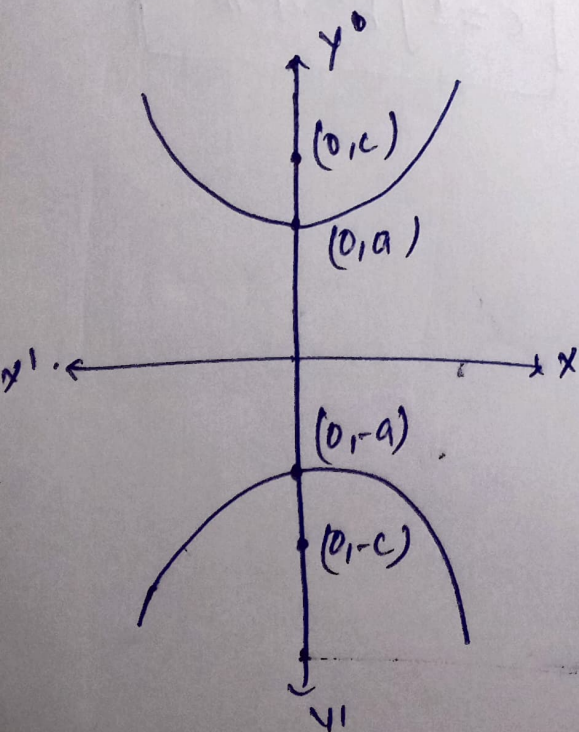
Standard Equation of Hyperbola

The simplest method to determine the equation of hyperbola is to assume the center of the hyperbola is at the origin $(0,0)$ and the foci lie either on x -axis or y -axis of the cartesian plane as shown below.



→ when foci lies on x -axis

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



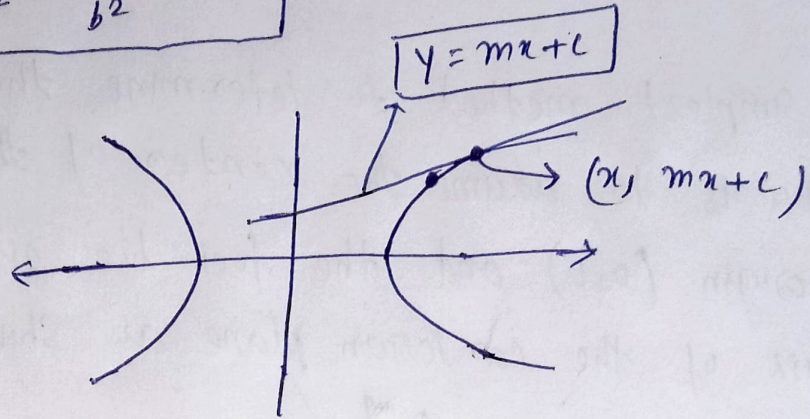
$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

What is the condition for a line $y = mx + c$ to be tangent of hyperbola

of hyperbola

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

Solⁿ



A line $y = mx + c$ becomes tangent to a curve when on solving the equations of the line and curve we get one solution.

Here solving these equation gives

$$\frac{x^2}{a^2} - \frac{(mx+c)^2}{b^2} = 1$$

solving above equation we get

$$\boxed{c^2 = b^2 - a^2 m^2} \quad \text{or} \quad \boxed{c = \pm \sqrt{b^2 - a^2 m^2}}$$

Equation of tangent at (x_1, y_1) on the Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

\Rightarrow The equation of the tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ at } (x_1, y_1) \text{ is}$$

$$\boxed{\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1}$$

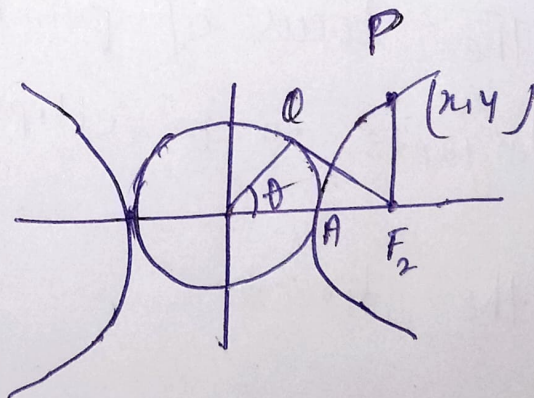
Find the locus of the point of intersection of orthogonal tangents to the Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

\Rightarrow The locus of points of intersection of perpendicular tangents to an hyperbola is director circle of that hyperbola. i.e. $x^2 + y^2 = a^2 - b^2$

So locus is $x^2 + y^2 = a^2 - b^2$

Parametric form of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

\Rightarrow The parametric co-ordinate of P are $(a \sec \theta, b \tan \theta)$



$$\begin{aligned} x &= a \sec \theta \\ y &= b \tan \theta \end{aligned}$$