Comprehension based questions-

The circle $x^2+y^2-8x=0$ and hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ intersect at the points A and B. ***(2010)***

4. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(a) $2x-\sqrt{5}y-20=0$ (b) $2x-\sqrt{5}y+4=0$

(c) $3x-4y+8=0$ (d) $4x-3y+4=0$

5. Equation of the circle with AB as its diameter is

(a) $x^2+y^2-12x+24=0$ (b) $x^2+y^2+12x+24=0$

(c) $x^2+y^2+24x-12=0$ (d) $x^2+y^2-24x-12=0$

Solution: -

4. **(b)** Any tangent to $\frac{x^2}{9}-\frac{y^2}{4}=1$ is $\frac{x\sec\alpha}{3}-\frac{y\tan\alpha}{2}=1$

It touches circle with center (4,0) and radius = 4

$$\therefore \frac{\frac{4\sec\alpha-3}{3}}{\sqrt{\frac{\sec^2\alpha}{9}+\frac{\tan^2\alpha}{4}}}=4$$

$$\Rightarrow 16\sec^2\alpha-24\sec\alpha+9=144\left(\frac{\sec^2\alpha}{9}+\frac{\tan^2\alpha}{4}\right)$$

$$\Rightarrow 12\sec^2\alpha+8\sec-15=0 \Rightarrow \sec\alpha=\frac{5}{6}\text{or}\frac{-3}{2}$$

but $\sec\alpha=\frac{5}{6}<1$ is not possible

$$\therefore \sec\alpha=-3/2 \Rightarrow \tan\alpha=\pm\frac{\sqrt{5}}{2}$$

$$\therefore \text{slope of tangent}=\frac{2\sec\alpha}{3\tan\alpha}=\frac{2(-3/2)}{3(-\sqrt{5}/2)}$$

(for +ve value of m)

$$=\frac{2}{\sqrt{5}}$$

$$\therefore \text{ Equation of tangent is } \frac{-x}{2}+\frac{y\sqrt{5}}{4}=1$$

or $2x-\sqrt{5}y+4=0$

5. **(a)** The intersection points of given circle

$$x^2 + y^2 - 8x = 0 \quad ...(1)$$

and hyperbola $4x^2 - 9y^2 - 36 = 0$ ...(2)

can be obtained by solving these equations Substituting value of y^2 from eqn (1) in eqn (2), we get

$$4x^2 - 9(8x - x^2) = 36 \Rightarrow 13x^2 - 72x - 36 = 0$$

$$\Rightarrow x = 6, \frac{-6}{13} \Rightarrow y^2 = 12, \frac{-48}{13} \frac{-36}{169} \text{ (not possible)}$$

$\therefore \left(6, 2\sqrt{3}\right)$ and $\left(6, -2\sqrt{3}\right)$ are points of intersection.

So eqn of required circle is

$$(x-6)(x-6) + \left(y - 2\sqrt{3}\right)\left(y + 2\sqrt{3}\right) = 0$$

$$\Rightarrow x^2 + 36 - 12x + y^2 - 12 = 0$$

$$\Rightarrow x^2 + y^2 - 12x + 24 = 0$$