

Question : - For what value of λ does the line $y = 2x + \lambda$ touches the hyperbola $16x^2 - 9y^2 = 144$?

Sol. Equation of hyperbola is

$$16x^2 - 9y^2 = 144$$

or
$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Comparing this with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get $a^2 = 9, b^2 = 16$

and comparing this line $y = 2x + \lambda$ with $y = mx + c$

$\therefore m = 2$ and $c = \lambda$

If the line $y = 2x + \lambda$ touches the hyperbola

$$16x^2 - 9y^2 = 144$$

then $c^2 = a^2m^2 - b^2$

$\Rightarrow \lambda^2 = 9(2)^2 - 16 = 36 - 16 = 20$

$\therefore \lambda = \pm 2\sqrt{5}$