VISHVAJEET KUMAR GOND, BS-MS(ECO), IIT KANPUR

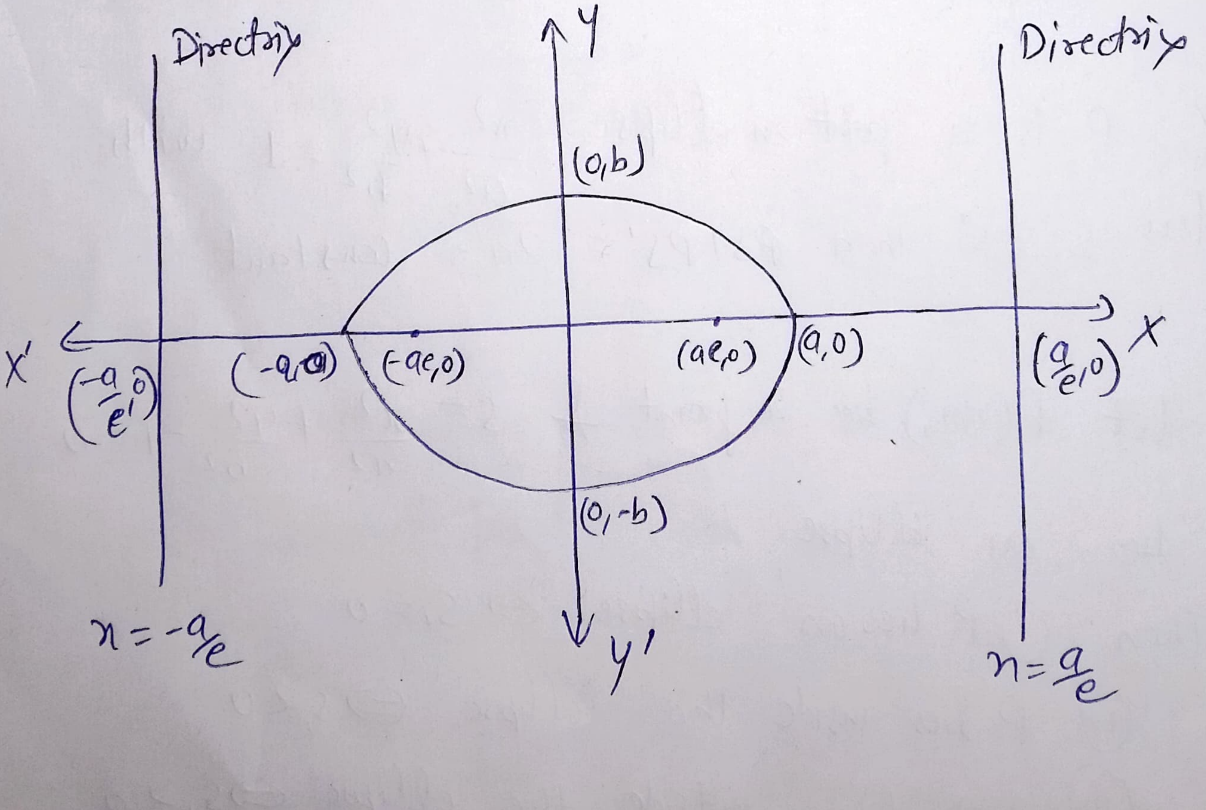
Notes - Ellipse

- Ellipse - Locus of a point which moves on a plane such that its distance from a fixed point is a constant ratio from a fixed line & this ratio, $e < 1$.

- Standard Eqⁿ of Ellipse -

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where $b^2 = a^2(1-e^2)$



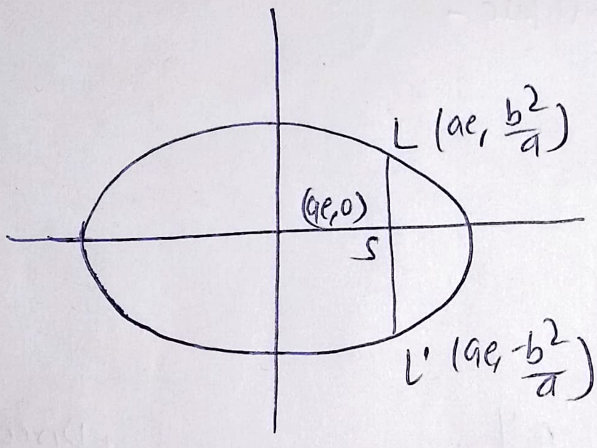
- Length of latus rectum:

(1) For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $\boxed{LR = \frac{2b^2}{a}}$

where $a > b$

(2) for ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $\boxed{LR = \frac{2a^2}{b}}$

where $0 < a < b$



- If P is a point on ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci S & S', then $PS + PS' = 2a$ = constant

- let $P(x_1, y_1)$ be a point & $S \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$ be an ellipse

Then, (i) P lies on ellipse $\Leftrightarrow S_1 = 0$

(ii) P lies inside the ellipse $\Leftrightarrow S_1 < 0$

(iii) P lies outside the ellipse $\Leftrightarrow S_1 > 0$

Intersection of a line and an Ellipse -

Eqⁿ of Ellipse ⇒ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

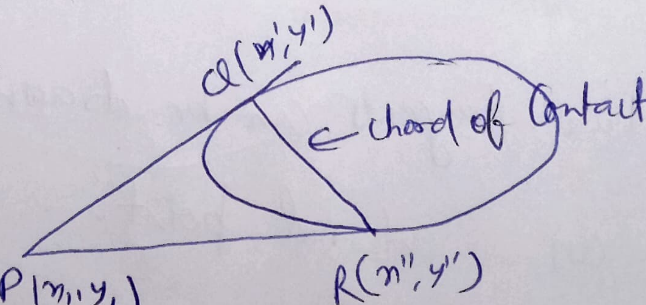
line ⇒ $y = mx + c$

Every straight line will cut the ellipse in two points may be real, coincident or imaginary according as,

$$\boxed{a^2m^2 + b^2 >, =, < c^2}$$

Chord of Contact -

The equation of chord of contact of tangents drawn from a point (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$



Note - This is the same as the eqⁿ of tangent but position of point differ.

- The equation of tangent to the ellipse $S=0$ at $P(x_1,y_1)$ is $S_1=0$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

- The equation of normal to the ellipse $S=0$ at $P(x_1,y_1)$ is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

- The condition that the line $y=mx+c$ may be a tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $c^2=a^2m^2+b^2$.

- <u>Eq^n of tangent</u> – m-form.

$$y=mx\pm\sqrt{a^2m^2+b^2}$$

where, point of contact $\left(-\frac{a^2m}{c},\frac{b^2}{c}\right)$

- Two tangents can be drawn to an ellipse from an external point.

Parametric Equation –

- If $P(x,y)$ is a point on the ellipse, then
  $x = a\cos\theta$, $y = b\sin\theta$, where θ is eccentric angle of P.

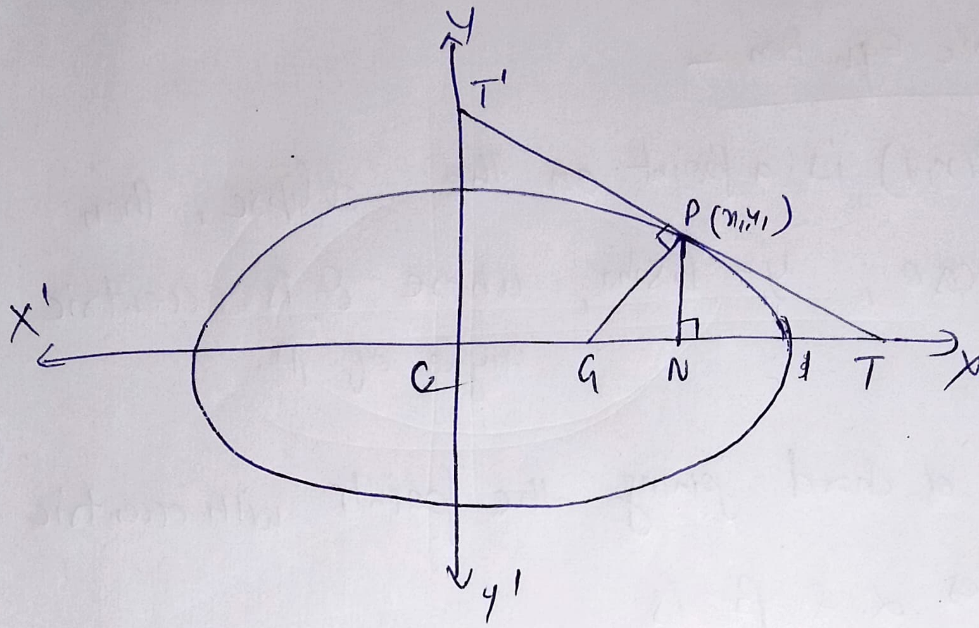
- Eq^n of chord joining the points with eccentric angles α & β is

$$\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$$

- Eq^n of tangent at $P(\theta)$ on ellipse $S=0$
  is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

- Eq^n of normal at $P(\theta)$ on ellipse $S=0$
  is $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2-b^2$

- Four normals can be drawn from any point to the ellipse & sum of the eccentric angles of their feet is an odd multiple of π.



- length of sub-tangent, $NT = CT - CN$

$$= \frac{a^2}{x_1} - x_1$$

- length of sub-normal, $GN = CN - CG$

$$= x_1 - \left(\frac{x_1}{1} - \frac{b^2}{a} x_1\right)$$

$$= \frac{b^2}{a^2} x_1 = (1-e^2)\, x_1$$