Q1 . Find the length of the line-segment joining the vertex of the parabola $y^2 = 4ax$ and a point on the parabola where line segment makes an angle θ to the x-axis.

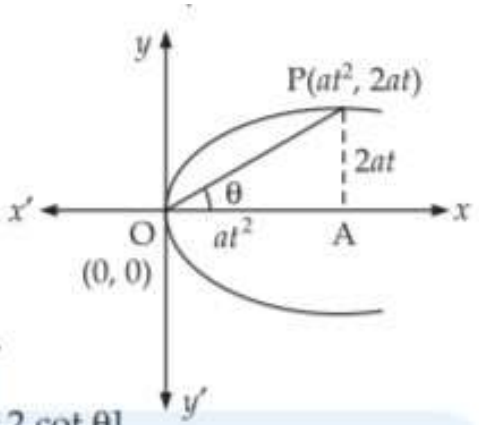
Sol. Equation of parabola is $y^2 = 4ax$

Let $P(at^2, 2at)$ be any point on the parabola.

In ΔPOA, we have

$$\tan\theta = \frac{2at}{at^2} = \frac{2}{t} \quad \Rightarrow \quad t = \frac{2}{\tan\theta}$$

$$\Rightarrow \quad t = 2\cot\theta \quad ...(i)$$



$$OP = \sqrt{(at^2 - 0)^2 + (2at - 0)^2}$$

$$= \sqrt{a^2t^4 + 4a^2t^2}$$

$$= at\sqrt{t^2 + 4}$$

$$= a \times 2\cot\theta\sqrt{4\cot^2\theta + 4}$$

$$[\because\ t = 2\cot\theta]$$

$$= 2a\cot\theta.2\sqrt{\cot^2\theta + 1} = 4a\cot\theta.\operatorname{cosec}\theta$$

$$= 4a.\frac{\cos\theta}{\sin\theta}.\frac{1}{\sin\theta} = \frac{4a\cos\theta}{\sin^2\theta}$$

Hence, the required length $= \dfrac{4a\cos\theta}{\sin^2\theta}$.