Q2 . Find the equation of each of the parabolas

(*i*) directrix = 0 and focus at (6, 0)

(*ii*) vertex at (0, 4), focus at (0, 2)

(*iii*) focus at (– 1, –2), directrix $x - 2y + 3 = 0$

Sol. (*i*) Given that directrix = 0 and focus (6, 0)

$\therefore$ The equation of the parabola is

$$(x-6)^2 + y^2 = x^2$$

$$\Rightarrow \quad x^2 + 36 - 12x + y^2 = x^2$$

$$\Rightarrow \quad y^2 - 12x + 36 = 0$$

Hence, the required equations is $y^2 - 12x + 36 = 0$

(*ii*) Given that vertex at (0, 4) and focus at (0, 2).

So, the equation of directrix is $y - 6 = 0$

According to the definition of the parabola

$$PF = PM.$$

$$\sqrt{(x-0)^2 + (y-2)^2} = |y-6|$$

$$\Rightarrow \sqrt{x^2 + y^2 + 4 - 4y} = |y-6|$$

Squaring both the sides, we get

$$x^2 + y^2 + 4 - 4y = y^2 + 36 - 12y$$

$$\Rightarrow \quad x^2 + 4 - 4y = 36 - 12y$$

$$\Rightarrow \quad x^2 + 8y - 32 = 0$$

$$\Rightarrow \quad x^2 = 32 - 8y$$

Hence, the required equation is $x^2 = 32 - 8y$.

(*iii*) Given that focus at (– 1, – 2) and directrix $x - 2y + 3 = 0$

Let (x, y) be any point on the parabola.

According to the definition of the parabola, we have

$$PF = PM$$

$$\sqrt{(x+1)^2+(y+2)^2} = \left|\frac{x-2y+3}{\sqrt{(1)^2+(-2)^2}}\right|$$

$$\Rightarrow \quad \sqrt{x^2+1+2x+y^2+4+4y} = \left|\frac{x-2y+3}{\sqrt{5}}\right|$$

Squaring both sides, we get

$$x^2+1+2x+y^2+4+4y = \frac{x^2+4y^2+9-4xy-12y+6x}{5}$$

$\Rightarrow 5x^2+5+10x+5y^2+20+20y = x^2+4y^2+9-4xy-12y+6x$

$\Rightarrow 4x^2+y^2+4xy+4x+32y+16=0$

Hence, the required equation is

$4x^2+4xy+y^2+4x+32y+16=0$