

Q2 . Find the equation of each of the parabolas

(i) directrix = 0 and focus at (6, 0)

(ii) vertex at (0, 4), focus at (0, 2)

(iii) focus at (-1, -2), directrix $x - 2y + 3 = 0$

Sol. (i) Given that directrix = 0 and focus (6, 0)

∴ The equation of the parabola is

$$(x - 6)^2 + y^2 = x^2$$

$$\Rightarrow x^2 + 36 - 12x + y^2 = x^2$$

$$\Rightarrow y^2 - 12x + 36 = 0$$

Hence, the required equations is $y^2 - 12x + 36 = 0$

(ii) Given that vertex at (0, 4) and focus at (0, 2).

So, the equation of directrix is $y - 6 = 0$

According to the definition of the parabola

$$PF = PM.$$

$$\sqrt{(x - 0)^2 + (y - 2)^2} = |y - 6|$$

$$\Rightarrow \sqrt{x^2 + y^2 + 4 - 4y} = |y - 6|$$

Squaring both the sides, we get

$$x^2 + y^2 + 4 - 4y = y^2 + 36 - 12y$$

$$\Rightarrow x^2 + 4 - 4y = 36 - 12y$$

$$\Rightarrow x^2 + 8y - 32 = 0$$

$$\Rightarrow x^2 = 32 - 8y$$

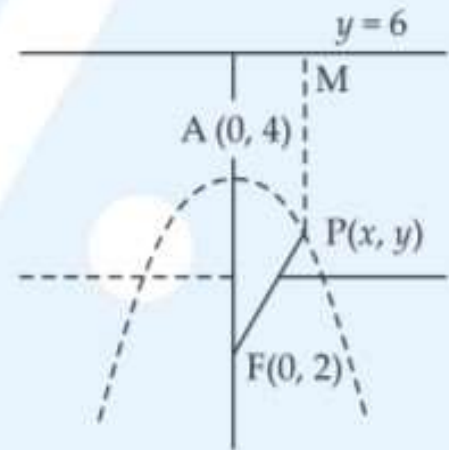
Hence, the required equation is $x^2 = 32 - 8y$.

(iii) Given that focus at (-1, -2) and directrix $x - 2y + 3 = 0$

Let (x, y) be any point on the parabola.

According to the definition of the parabola, we have

$$PF = PM$$



$$\sqrt{(x+1)^2 + (y+2)^2} = \left| \frac{x-2y+3}{\sqrt{(1)^2 + (-2)^2}} \right|$$

$$\Rightarrow \sqrt{x^2 + 1 + 2x + y^2 + 4 + 4y} = \left| \frac{x-2y+3}{\sqrt{5}} \right|$$

Squaring both sides, we get

$$x^2 + 1 + 2x + y^2 + 4 + 4y = \frac{x^2 + 4y^2 + 9 - 4xy - 12y + 6x}{5}$$

$$\Rightarrow 5x^2 + 5 + 10x + 5y^2 + 20 + 20y = x^2 + 4y^2 + 9 - 4xy - 12y + 6x$$

$$\Rightarrow 4x^2 + y^2 + 4xy + 4x + 32y + 16 = 0$$

Hence, the required equation is

$$4x^2 + 4xy + y^2 + 4x + 32y + 16 = 0$$