Q)

If $y(t)$ is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$, then show that $y(1) = -\frac{1}{2}$.

Soln :

Given that, $$(1+t)\frac{dy}{dt} - ty = 1$$

$$\Rightarrow \quad \frac{dy}{dt} - \left(\frac{t}{1+t}\right) y = \frac{1}{1+t}$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dt} + Py = Q$, we get

$$P = -\left(\frac{t}{1+t}\right), Q = \frac{1}{1+t}$$

$$\text{IF} = e^{-\int \frac{t}{1+t} dt} = e^{-\int \left(1 - \frac{1}{1+t}\right) dt} = e^{-[t - \log(1+t)]}$$

$$= e^{-t} \cdot e^{\log(1+t)}$$

$$= e^{-t}(1+t)$$

The general solution is

$$y(t) \cdot \frac{(1+t)}{e^t} = \int \frac{(1+t) \cdot e^{-t}}{(1+t)} dt + C$$

$$\Rightarrow \quad y(t) = \frac{e^{-t}}{(-1)} \cdot \frac{e^t}{1+t} + C', \text{ where } C' = \frac{C e^t}{1+t}$$

$$\Rightarrow \quad y(t) = -\frac{1}{1+t} + C'$$

When $t = 0$ and $y = -1$, then

$$-1 = -1 + C' \Rightarrow C' = 0$$

$$y(t) = -\frac{1}{1+t} \Rightarrow y(1) = -\frac{1}{2}$$