Q)

Find the general solution of $(x + 2y^3)\frac{dy}{dx} = y.$

Soln :

Given that, $(x + 2y^3)\frac{dy}{dx} = y$

$\Rightarrow$ $y \cdot \frac{dx}{dy} = x + 2y^3$

$\Rightarrow$ $\frac{dx}{dy} = \frac{x}{y} + 2y^2$

$\Rightarrow$ $\frac{dx}{dy} - \frac{x}{y} = 2y^2$

which is a linear differential equation.

On comparing it with $\frac{dx}{dy} + Px = Q$, we get

$P = -\frac{1}{y}, Q = 2y^2$

$\text{IF} = e^{\int -\frac{1}{y} dy} = e^{-\int \frac{1}{y} dy}$

$\therefore$ $= e^{-\log y} = \frac{1}{y}$

The general solution is $x \cdot \frac{1}{y} = \int 2y^2 \cdot \frac{1}{y} dy + C$

$\Rightarrow$ $\frac{x}{y} = \frac{2y^2}{2} + C$

$\Rightarrow$ $\frac{x}{y} = y^2 + C$

$\Rightarrow$ $x = y^3 + Cy$