Q)

Solve $ydx - xdy = x^2 ydx$.

Soln :

Given that, $$ydx - xdy = x^2 ydx$$

$$\Rightarrow \quad \frac{1}{x^2} - \frac{1}{xy}.\frac{dy}{dx} = 1$$

$$\Rightarrow \quad -\frac{1}{xy}.\frac{dy}{dx} + \frac{1}{x^2} - 1 = 0$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{xy}{x^2} + xy = 0$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} + xy = 0$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(x - \frac{1}{x}\right)y = 0$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \left(x - \frac{1}{x}\right), Q = 0$$

$$\text{IF} = e^{\int Pdx}$$

$$= e^{\int\left(x - \frac{1}{x}\right)dx}$$

$$= e^{\frac{x^2}{2} - \log x}$$

$$= e^{\frac{x^2}{x}}, e^{-\log x}$$

$$= \frac{1}{x}e^{\frac{x^2}{2}}$$

The general solution is

$$y \cdot \frac{1}{x}e^{x^2/2} = \int 0 \cdot \frac{1}{x}e^{x^2/2}\, dx + C$$

$$\Rightarrow \quad y \cdot \frac{1}{x}e^{x^2/2} = C$$

$$\Rightarrow \quad y = C\, x\, e^{-x^2/2}$$