

Q) The orthogonal trajectories for the family of curves $r^2 = a^2 \cos 4\theta$ are:

Soln :

We have, $r^2 = a^2 \cos 4\theta = a^2 (1 - 2 \sin^2 2\theta)$

Differentiating w.r.t. θ , we get

$$2r \frac{dr}{d\theta} = -4a^2 \sin 4\theta \quad \dots(2)$$

Eliminating a from (2) using (1), we get

$$\frac{2}{r} \frac{dr}{d\theta} = -\frac{4 \sin 4\theta}{\cos 4\theta} \quad \dots(3)$$

Replacing $\frac{dr}{d\theta}$ with $-r^2 \frac{d\theta}{dr}$ in (3), we get

$$2r \frac{d\theta}{dr} = \frac{4 \sin 4\theta}{\cos \theta}$$
$$\Rightarrow \frac{2}{r} dr = \frac{\cos \theta}{\sin \theta} d\theta$$

Integrating, we get

$$2 \log r = \frac{1}{4} \log \sin 4\theta + 2 \log c$$
$$\Rightarrow r^8 = c^8 \sin 4\theta$$