Past Year JEE Questions

Questions

Quetion: 01

If P and Q are the points of intersection of the circles
$x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$ then there is a circle passing through P, Q and $(1, 1)$ for:
A. all except one value of p
B. all except two values of p
C. exactly one value of p
D. all values of p

Solutions

Solution: 01

Explanation

The given circles are

$S_1 \equiv x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ $\ldots (1)$

$S_2 \equiv x^2 + y^2 + 2x + 2y - p^2 = 0$ $\ldots (2)$

$\therefore$ Equation of common chord PQ is $S_1 - S_2 = 0$

$\Rightarrow L \equiv x + 5y + p^2 + 2p - 5 = 0$

$\Rightarrow$ Equation of circle passing through P and Q is

$S_1 + \lambda \, L = 0$

$\Rightarrow \left(x^2 + y^2 + 3x + 7y + 2p - 5 \right) + \lambda$

$\left(x + 5y + p^2 + 2p - 5 \right) = 0$

As it passes through $(1, 1)$, therefore

$\Rightarrow (7 + 2p) + \lambda \left(2p + p^2 + 1 \right) = 0$

$\Rightarrow \lambda = -\dfrac{2p + 7}{(p + 1)}$

which does not exist for $p = -1$