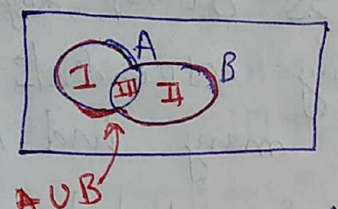
Math class 11th, chapter 1:
Principle of inclusion and exclusion.

Lecture-1

Example of problem where we can use principle of inclusion exclusion.
we say in sort (PIE) (Principle of inclusion exclusion)

let say A, B finite sets
$A \cup B$, $A \cap B$, take cardinality $|A \cup B| + |A \cap B| = |A| + |B|$ (*)

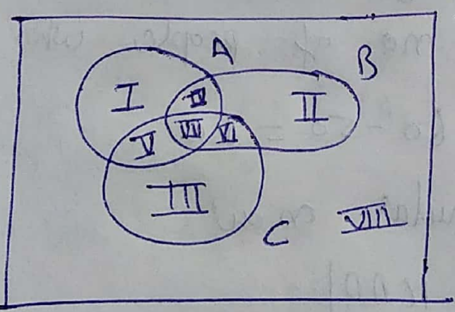
we have Venn diagram (given in NCERT)



X (universal set)
*compliment of set have no mean if universal set is not given

$A \cup B$ = all element in the region I + II + III.
$A \cap B$ = region III.
A = I and III
B = II and III
so we have geometric represantation formula (*).

* 3 set



each region are consider to be a set.
so A consist of regions.
$A = I + IV + V + VII$
$B = II + IV + VI + VII$
$C = III + V + VI + VIII$

we know, as proved above
$|A \cup B| + |A \cap B| = |A| + |B|$
$\rightarrow |A \cup B| = |A| + |B| - |A \cap B|$ (Direct)
lets look $|X / (A \cup B)| = (A \cup B)^c$ (compliment of set)
$= |X| - |A \cup B|$

$|(A \cup B)^c| = |X| - (|A| + |B| - |A \cap B|)$
$= |X| - (|A| + |B|) + |A \cap B|$

so lets I have 3 sets A, B, C,
so $|A \cup B \cup C| = |A| + |B| + |C| - (|A \cap B| + |A \cap C| + |B \cap C|) + |A \cap B \cap C|$
(Direct union)

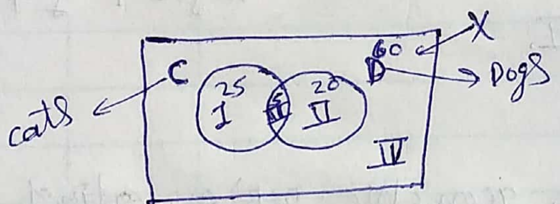
lets try to find.

$|X - (A \cup B \cup C)| = |X| - |A \cup B \cup C|$

$|(A \cup B \cup C)^c| = |X| - (|A| + |B| + |C|) + (|A \cap B| + |A \cap C| + |B \cap C|)$
$- (|A \cap B \cap C|)$ (compliment)

problem 1. Among 60 pet owners responding to a survey, 30 owned cats, 25 owned dogs, and 5 owned both. How many respondents had a cat or a dog? How many had neither?

solution – lets draw venn diagram to look geometrically



$|C \cup D| = \underline{25 + 5 + 20} = 50$ so IV = 10

so either having dogs cats or both to 50 people.
we want those no of people who have not both $|X - (C \cup D)| = 60 - 50 = 10$

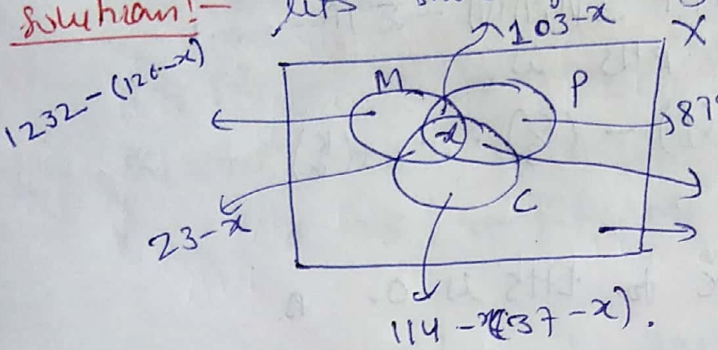
lets do by formulas now

$|C \cup D| = |C| + |D| - |C \cap D|$
$= 30 + 25 - 5$
$= 50$

$|X - (C \cup D)| = |X| - |C \cup D|$
$= 60 - 50$
$= 10$

Problem 2. A total of 1232 students have taken M, 879 have taken P, 114 have taken C, further 103 have taken both M & P, 23 have taken both M & C and 14 have taken both P & C. If 2092 students have taken at least one of P, C, M, how many have taken all three?

Solution:- lets solve first by venn diagram.



we will follow back track, to find all region value.

adding algebraically this seven number and equating to 2092 to find x.

lets do directly by formula itself for 3 sets.

$$|A \cup B \cup C| = |A| + |B| + |C| - (|A \cap B| + |A \cap C| + |B \cap C|) + (|A \cap B \cap C|).$$

$\Rightarrow$ let A stand for M

and B for P

and C for C

$$\Rightarrow 2092 = 1232 + 879 + 114 - (103 + 23 + 14) + x$$

$$\Rightarrow x = 7.$$

*** (PIE) complement version)**

let X be a finite set, and let $A_1, A_2, \ldots A_n$ be non empty subsets of X, then

$$|X \setminus \bigcup_{j=1}^{n} A_j| = \sum_{I \subseteq [n]} (-1)^{|I|} |A_I|$$ where $A_I := \bigcap_{i \in I} A_i$

we use $A_\phi := X$

where $[n] = \{1, 2, 3, \ldots n\}$.

Proof:- $|X - \bigcup_{i=1}^{n} A_i| = \sum_{I \subseteq [n]} (-1)^{|I|} |A_I|$,

where $A_I = \bigcap_{i \in I} A_i \quad (A_\phi = X)$

so let $x \in X$, contributes +1 to LHS.

case 1: $x \notin \bigcup_{i=} A_i$,

$$RHS = \underset{+1}{|X|} - \sum \underset{0}{|A_i|} + \sum \underset{0}{|A_i \cap A_j|} - \cdots + \cdots + \cdots$$

Case 2: $x \in \bigcup_{i=1}^{n} A_i$, suppose x belongs to exactly k of $A_1, A_2, \ldots A_n$ $(1 \le k \le n)$, for simplicity suppose $x \in A_1, A_2, \ldots A_k$;

$$RHS = |X| - \sum |A_i| + \sum |A_i \cap A_j| - \sum |A \ldots \ldots + \cdots$$

contribution of x to RHS is

$$= 1 - k + \binom{k}{2} - \binom{k}{3} \cdots \pm \binom{k}{k}$$

$$= (1-1)^k$$

$$= 0$$

contribution of x to LHS is 0.