At the moment $t = 0$ the force $F = at$ is applied to a small body of mass m resting on a smooth horizontal plane (a is a constant).

The permanent direction of this force forms an angle α with the horizontal (Fig. 1.14). Find:

(a) the velocity of the body at the moment of its breaking off the plane;

(b) the distance traversed by the body up to this moment.

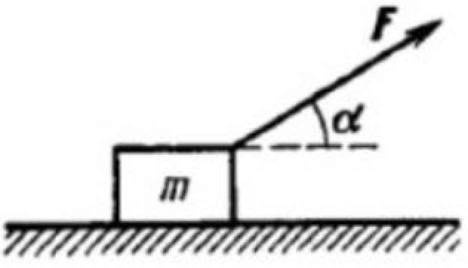
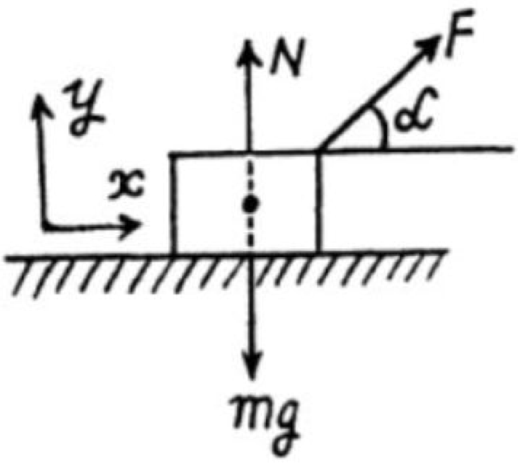


Fig. 1.14.

First of all let us draw the free body diagram for the small body of mass m and indicate x-axis along the horizontal plane and y-axis, perpendicular to it, as shown in the figure. Let the block breaks off the plate at $t=t_0$ i.e. N=0



So, $N = mg - at_0 \sin\alpha = 0$

or, $t_0 = \dfrac{mg}{a\sin\alpha}$ (1)

From $F_x = m\,w_x$, for the body under investigation :

$m\,dv/dt = at\cos\alpha$; Integrating within the limits for $v(t)$

$$m\int_0^v dv_x = a\cos\alpha\int_0^t t\,dt \quad \text{(using Eq. 1)}$$

So, $$v = \frac{ds}{dt} = \frac{a\cos\alpha}{2m}t^2 \quad (2)$$

Integrating, Eqn. (2) for $s(t)$

$$s = \frac{a\cos\alpha}{2m}\frac{t^3}{3} \quad (3)$$

Using the value of $t = t_0$

from Eq. (1), into Eqs. (2) and (3)

$$v = \frac{mg^2\cos\alpha}{2a\sin^2\alpha} \text{ and } s = \frac{m^2g^3\cos\alpha}{6a^2\sin^3\alpha}$$