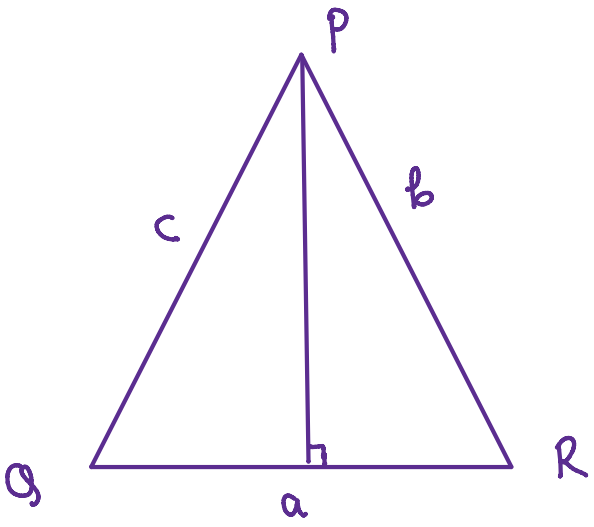
Question. If in a ΔPQR, $\sin P$, $\sin Q$ and $\sin R$ are in A.P., then the altitudes are in ?

Solution.



Let p_1, p_2, p_3 be altitudes from P, Q, R

Now,

Area of triangle $\Delta = \frac{1}{2} p_1 \times QR = \frac{1}{2} p_2 \times PR = \frac{1}{2} p_3 \times PQ$

$\Rightarrow$ $p_1 = \frac{2\Delta}{a}$, $p_2 = \frac{2\Delta}{b}$, $p_3 = \frac{2\Delta}{c}$ —— (1)

Now, by Sine Law:→

$$\frac{a}{\sin P} = \frac{b}{\sin Q} = \frac{c}{\sin R} = k$$

$\Rightarrow$ $a = k \sin P$, $b = k \sin Q$, $c = k \sin R$

$\Rightarrow$ $\frac{2\Delta}{p_1} = k \sin P$, $\frac{2\Delta}{p_2} = k \sin Q$, $\frac{2\Delta}{p_3} = k \sin R$

$$\Rightarrow \quad P_1 = \left(\frac{2\Delta}{k}\right)\frac{1}{\sin P},$$

$$P_2 = \left(\frac{2\Delta}{k}\right)\cdot\frac{1}{\sin Q},$$

$$P_3 = \left(\frac{2\Delta}{k}\right)\cdot\frac{1}{\sin R}$$

Now, since $\sin P, \sin Q$ and $\sin R$ are in A.P.

$\therefore$ P_1, P_2, P_3 are in H.P. Ans.