Question 5

Question. If in ΔABC, the distances of the vertices from the orthocentre are x, y and z, then prove that

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = \frac{abc}{xyz}$$

Solution:→

We know that:→

Distance of orthocentre (H) from vertex (A) = $2R\cos A$

Also, Sides of triangle = $2R\sin A$, $2R\sin B$, $2R\sin C$

$$\therefore \quad \frac{a}{x} + \frac{b}{y} + \frac{c}{z}$$

$$= \frac{2R\sin A}{2R\cos A} + \frac{2R\sin B}{2R\cos B} + \frac{2R\sin C}{2R\cos C}$$

$$= \tan A + \tan B + \tan C \quad \text{——— (1)}$$

Now, $A + B + C = 180^\circ$

$$\Rightarrow \quad \tan(A+B+C) = \tan 180^\circ$$

$$\Rightarrow \quad \frac{\tan A + \tan B + \tan C - \tan A \cdot \tan B \cdot \tan C}{1 - \tan A \cdot \tan B - \tan B \cdot \tan C - \tan C \cdot \tan A} = 0$$

$$\Rightarrow \quad \tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$

$\therefore$ From equation (1):→

$\therefore$ From equation (1):$\rightarrow$

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = \tan A + \tan B + \tan C$$

$$= \tan A \cdot \tan B \cdot \tan C$$

$$= \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B} \cdot \frac{\sin C}{\cos C}$$

$$= \frac{(2R \sin A)}{(2R \cos A)} \cdot \frac{(2R \sin B)}{(2R \cos B)} \cdot \frac{(2R \sin C)}{(2R \cos C)}$$

$$= \frac{abc}{xyz}$$

Hence, proved