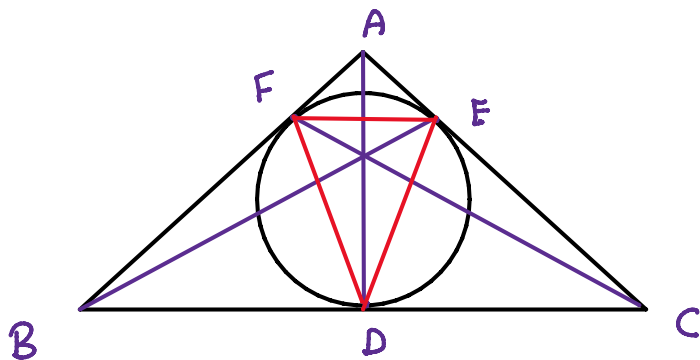Question. In a triangle ABC, AD, BE and CF are the altitudes and R is the circumradius, then the radius of the circle DEF is:→

(a) 2R

(b) R

(c) R/2

(d) None of these

Solution.

Clearly $\triangle DEF$ is pedal triangle.

$\therefore$ $EF = a \cos A$, $\angle EDF = 180° - 2A$

$DE = c \cos C$

$DF = b \cos B$

Let R_1 is the circumradius of $\triangle DEF$

$\therefore$ $R_1 = \dfrac{EF \times \cancel{DE} \times \cancel{DF}}{4 \times \frac{1}{2} \times \cancel{DF} \times \cancel{DE} \times \sin \angle EDF}$

$\left\{ \because \text{ Circumradius} = \dfrac{abc}{4\Delta} \right\}$

$= \dfrac{EF}{2 \sin \angle EDF)}$

$= \dfrac{a \cos A}{2 \times \sin(180 - 2A)}$

$$= \frac{a \cos A}{2 \sin 2A}$$

$$= \frac{(2R \sin A) \cos A}{2 \sin 2A}$$

$\{ \because a = 2R \sin A \}$

$$= \frac{R \sin 2A}{2 \sin 2A}$$

$\boxed{R_1 = R/2}$ Ans Option (C)