

Question. ABC is an isosceles triangle inscribed in a circle of radius ' r '. If $AB = AC$ and h is the altitude from A to BC, then the ΔABC has

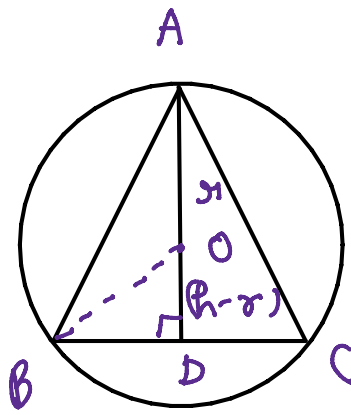
area $A = ?$

perimeter $P = ?$

$$\lim_{h \rightarrow 0} \frac{A}{P^3} = ?$$

[IIT 1989]

Solution.



$\therefore O$ is the circumcentre

$$\therefore OA = OB = OC = r$$

$$\text{and } BD = CD$$

$$\text{In } \Delta OBD, \quad BD = \sqrt{OB^2 - OD^2}$$

$$= \sqrt{r^2 - (h-r)^2}$$

$$= \sqrt{2hr - h^2}$$

$$\therefore BC = 2\sqrt{2hr - h^2} \quad \text{--- (1)}$$

\therefore In ΔABD

$$AB = AC = \sqrt{AD^2 + BD^2}$$

$$= \sqrt{h^2 + 2hr - h^2}$$

$$= \sqrt{\cancel{h^2} + 2hr - \cancel{h^2}}$$

$$= \sqrt{2hr} \quad \text{--- (11)}$$

$$\therefore \text{Perimeter} = AB + AC + BC$$

$$= \sqrt{2hr} + \sqrt{2hr} + 2\sqrt{2hr - h^2}$$

$$P = 2(\sqrt{2hr - h^2} + \sqrt{2hr}) \quad \underline{\underline{\text{Ans.}}}$$

$$\therefore \text{Area} = \frac{1}{2} \times BC \times AD$$

$$= \frac{1}{2} \times \cancel{2} \sqrt{2hr - h^2} \times h$$

$$\Rightarrow \quad A = h \sqrt{2hr - h^2} \quad \underline{\underline{\text{Ans.}}}$$

$$\therefore \lim_{h \rightarrow 0} \frac{A}{P^3} = \lim_{h \rightarrow 0} \frac{h \sqrt{2hr - h^2}}{\{2[\sqrt{2hr - h^2} + \sqrt{2hr}]\}^3}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}^{3/2} \sqrt{2r - h}}{\cancel{8} \cancel{h}^{3/2} [\sqrt{2r - h} + \sqrt{2r}]^3}$$

$$= \frac{1}{8} \times \frac{\sqrt{2r}}{\{2\sqrt{2r}\}^3}$$

$$= \frac{1}{8} \times \frac{\sqrt{2r}}{8 \times 2\sqrt{2} \times r^{3/2}}$$

$$\lim_{h \rightarrow 0} \frac{A}{p^3} = \frac{1}{128 \pi}$$

Ans