

$$\text{c) } \lim_{x \rightarrow -\infty} \frac{x^3 - 9x + 1}{3x^2 - 2x - 15} = -\infty$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{x^3 - 9x + 1}{3x^2 - 2x - 15} &= \lim_{x \rightarrow -\infty} \frac{x^3 \left(1 - \frac{9}{x^2} + \frac{1}{x^3} \right)}{3x^2 \left(1 - \frac{2}{3x} - \frac{5}{x^2} \right)} = \lim_{x \rightarrow -\infty} \frac{x^3}{3x^2} \cdot \lim_{x \rightarrow -\infty} \frac{1 - \frac{9}{x^2} + \frac{1}{x^3}}{\left(1 - \frac{2}{3x} - \frac{5}{x^2} \right)} \\ &= \left(\lim_{x \rightarrow -\infty} \frac{x^3}{3x^2} \right) \cdot 1 = \left(\lim_{x \rightarrow -\infty} \frac{x}{3} \right) \cdot 1 = -\infty \cdot 1 = -\infty \end{aligned}$$