4. Compute each of the following limits.

a) $\lim_{x \to -\infty} \dfrac{x + x^2 - 6}{6x + 5x^2 + 2x^3} = 0$

Solution: The numerator approaches infinity and the denominator approaches negative infinity. This does not give us enough information about the quotient. A limit like this is called an **indeterminate**. We will bring this expression to a form that is not an indeterminate. Let us rearrange the polynomials in the rational function given. Then we will factor out the leading term in the numerator and denominator.

$$\lim_{x \to -\infty} \frac{x^2 + x - 6}{2x^3 + 5x^2 + 6x} = \lim_{x \to -\infty} \frac{x^2\left(1 + \dfrac{1}{x} - \dfrac{6}{x^2}\right)}{2x^3\left(1 + 5 + \dfrac{6}{x}\right)}$$

We now express the limit of the product as the product of two limits

$$\lim_{x \to -\infty} \frac{x^2\left(1 + \dfrac{1}{x} - \dfrac{6}{x^2}\right)}{2x^3\left(1 + 5 + \dfrac{6}{x}\right)} = \lim_{x \to -\infty} \frac{x^2}{2x^3} \cdot \lim_{x \to -\infty} \frac{\left(1 + \dfrac{1}{x} - \dfrac{6}{x^2}\right)}{\left(1 + 5 + \dfrac{6}{x}\right)}$$

The first expression can be simplified and thus has a limit we can easily determine its limit. The second expression, although looks unfriendly, is always going to approach 1.

$$\lim_{x \to -\infty} \frac{x^2}{2x^3} \cdot \lim_{x \to -\infty} \frac{\left(1 + \dfrac{1}{x} - \dfrac{6}{x^2}\right)}{\left(1 + 5 + \dfrac{6}{x}\right)} = \lim_{x \to -\infty} \frac{1}{2x} \cdot 1 = 0 \cdot 1 = 0$$

The entire computation should look like this:

$$\begin{aligned}\lim_{x \to -\infty} \frac{x^2 + x - 6}{2x^3 + 5x^2 + 6x} &= \lim_{x \to -\infty} \frac{x^2\left(1 + \dfrac{1}{x} - \dfrac{6}{x^2}\right)}{2x^3\left(1 + 5 + \dfrac{6}{x}\right)} = \lim_{x \to -\infty} \frac{x^2}{2x^3} \cdot \lim_{x \to -\infty} \frac{\left(1 + \dfrac{1}{x} - \dfrac{6}{x^2}\right)}{\left(1 + 5 + \dfrac{6}{x}\right)} \\ &= \lim_{x \to -\infty} \frac{1}{2x} \cdot 1 = 0 \cdot 1 = 0\end{aligned}$$