**Illustration 74** The value of $\lim_{n \to \infty} \left( \frac{a - 1 + \sqrt[n]{b}}{a} \right)^n$ $(a > 0, b > 0)$ is equal to

(a) $\sqrt[a]{b}$ (b) $\sqrt[b]{a}$

(c) $\sqrt{b}$ (d) $\sqrt{a}$

***Solution.*** Here, $\lim_{n \to \infty} \left( \frac{a - 1 + \sqrt[n]{b}}{a} \right)^n$ ($1^\infty$ form

$$= \lim_{n \to \infty} \left( 1 + \frac{\sqrt[n]{b} - 1}{a} \right)^n = e^{\lim_{n \to \infty} \left( \frac{\sqrt[n]{b} - 1}{a} \right) \cdot n}$$

$$= e^{\lim_{n \to \infty} \frac{1}{a} \cdot \frac{b^{1/n} - 1}{1/n}} = e^{\lim_{n \to \infty} \frac{1}{a} \cdot \frac{b^{1/n} \log b \left( \frac{-1}{n^2} \right)}{-1/n^2}}$$

$$= e^{\frac{1}{a} \log_e b} = e^{\log_e b^{1/a}} = b^{1/a}$$

Hence, (a) is the correct answer.