

**Illustration 68** Solve  $\lim_{n \rightarrow \infty} \frac{-3n + (-1)^n}{4n - (-1)^n}$ .

**Solution.** Here,  $\lim_{n \rightarrow \infty} \frac{-3n + (-1)^n}{4n - (-1)^n}$ , two cases arise,  $n$  is even or  $n$  is odd.

**Case I**  $n$  is even, say  $n = 2k$ .

$$\lim_{k \rightarrow \infty} \frac{-6k + (-1)^{2k}}{8k - (-1)^{2k}}$$

$$= \lim_{k \rightarrow \infty} \frac{-6k + 1}{8k - 1} = \lim_{k \rightarrow \infty} \frac{-6 + \frac{1}{k}}{8 - \frac{1}{k}}$$

$$= \frac{-6}{8} = \frac{-3}{4}$$

(As  $\frac{1}{k} \rightarrow 0$ , when  $k \rightarrow \infty$ )

**Case II**  $n$  is odd, say  $n = 2k + 1$ .

$$\lim_{k \rightarrow \infty} \frac{-3(2k + 1) + (-1)^{2k+1}}{4(2k + 1) - (-1)^{2k+1}}$$

$$= \lim_{k \rightarrow \infty} \frac{-6k - 3 - 1}{8k + 4 + 1} = \lim_{k \rightarrow \infty} \frac{-6 - \frac{4}{k}}{8 + \frac{5}{k}} = \frac{-3}{4}$$

$\therefore$

$$\lim_{n \rightarrow \infty} \frac{-3n + (-1)^n}{4n - (-1)^n} = \frac{-3}{4}$$