

Illustration 63 Evaluate $\lim_{x \rightarrow \infty} \left(\frac{x+6}{x+1} \right)^{x+4}$.

Solution. As $x \rightarrow \infty$, $\lim_{x \rightarrow \infty} \frac{x+6}{x+1} = 1$ and $(x+4) \rightarrow \infty$

$$\begin{aligned} \therefore \lim_{x \rightarrow \infty} \left(\frac{x+6}{x+1} \right)^{x+4} &= \lim_{x \rightarrow \infty} \left[1 + \left(\frac{x+6}{x+1} - 1 \right) \right]^{x+4} = \lim_{x \rightarrow \infty} \left[1 + \frac{5}{x+1} \right]^{x+4} \\ &= e^{\lim_{x \rightarrow \infty} \left(\frac{5}{x+1} \right) \cdot (x+4)} = e^{\lim_{x \rightarrow \infty} 5 \cdot \frac{x+4}{x+1}} \\ &= e^{5 \cdot (1)} \\ &= e^5 \end{aligned} \quad \left[\text{As } x \rightarrow \infty; \lim_{x \rightarrow \infty} \frac{x+4}{x+1} = 1 \right]$$