11.8 Wave Nature of Matter

The dual (wave-particle) nature of light (electromagnetic radiation, in general) comes out clearly from what we have learnt in this and the preceding chapters. The wave nature of light shows up in the phenomena of interference, diffraction and polarisation. On the other hand, in photoelectric effect and Compton effect which involve energy and momentum transfer, radiation behaves as if it is made up of a bunch of particles – the photons. Whether a particle or wave description is best suited for understanding an experiment depends on the nature of the experiment. For example, in the familiar phenomenon of seeing an object by our eye, both descriptions are important. The gathering and focussing mechanism of light by the eye-lens is well described in the wave picture. But its absorption by the rods and cones (of the retina) requires the photon picture of light.

A natural question arises: If radiation has a dual (wave-particle) nature, might not the particles of nature (the electrons, protons, etc.) also exhibit wave-like character? In 1924, the French physicist Louis Victor de Broglie (pronounced as de Broy) (1892-1987) put forward the bold hypothesis that moving particles of matter should display wave-like properties under suitable conditions. He reasoned that nature was symmetrical and that the two basic physical entities – matter and energy, must have symmetrical character. If radiation shows dual aspects, so should matter. De Broglie proposed that the wave length λ associated with a particle of momentum p is given as

$$\lambda = \frac{h}{p} = \frac{h}{mv} \tag{11.5}$$

where m is the mass of the particle and v its speed. Equation (11.5) is known as the *de Broglie relation* and the wavelength λ of the *matter wave* is called *de Broglie wavelength*. The dual aspect of matter is evident in the de Broglie relation. On the left hand side of Eq. (11.5), λ is the attribute of a wave while on the right hand side the momentum p is a typical attribute of a particle. Planck's constant h relates the two attributes.

Equation (11.5) for a material particle is basically a hypothesis whose validity can be tested only by experiment. However, it is interesting to see that it is satisfied also by a photon. For a photon, as we have seen,

$$p = h\nu / c \tag{11.6}$$

Therefore,

$$\frac{h}{p} = \frac{c}{\nu} = \lambda \tag{11.7}$$

That is, the de Broglie wavelength of a photon given by Eq. (11.5) equals the wavelength of electromagnetic radiation of which the photon is a quantum of energy and momentum.

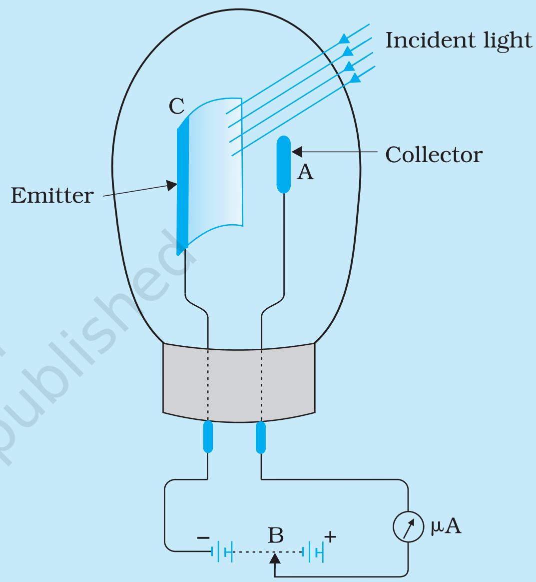
Clearly, from Eq. (11.5), λ is smaller for a heavier particle (large m) or more energetic particle (large v). For example, the de Broglie wavelength of a ball of mass 0.12 kg moving with a speed of 20 m s^{-1} is easily calculated:

PHOTOCELL

A photocell is a technological application of the photoelectric effect. It is a device whose electrical properties are affected by light. It is also sometimes called an electric eye. A photocell consists of a semi-cylindrical photo-sensitive metal plate C (emitter) and a wire loop A (collector) supported in an evacuated glass or quartz bulb. It is connected to the external circuit having a high-tension battery B and microammeter (μA) as shown in the Figure. Sometimes, instead of the plate C, a thin layer of photosensitive material is pasted on the inside of the bulb. A part of the bulb is left clean for the light to enter it.

When light of suitable wavelength falls on the emitter C, photoelectrons are emitted. These photoelectrons are drawn to the collector A. Photocurrent of the order of a few microampere can be normally obtained from a photo cell.

A photocell converts a change in intensity of illumination into a change in photocurrent. This current can be used to operate control systems and in light measuring devices. A photocell of lead sulphide sensitive to infrared radiation is used in electronic ignition circuits.



A photo cell

In scientific work, photo cells are used whenever it is necessary to measure the intensity of light. Light meters in photographic cameras make use of photo cells to measure the intensity of incident light. The photocells, inserted in the door light electric circuit, are used as automatic door opener. A person approaching a doorway may interrupt a light beam which is incident on a photocell. The abrupt change in photocurrent may be used to start a motor which opens the door or rings an alarm. They are used in the control of a counting device which records every interruption of the light beam caused by a person or object passing across the beam. So photocells help count the persons entering an auditorium, provided they enter the hall one by one. They are used for detection of traffic law defaulters: an alarm may be sounded whenever a beam of *(invisible)* radiation is intercepted.

In burglar alarm, (invisible) ultraviolet light is continuously made to fall on a photocell installed at the doorway. A person entering the door interrupts the beam falling on the photocell. The abrupt change in photocurrent is used to start an electric bell ringing. In fire alarm, a number of photocells are installed at suitable places in a building. In the event of breaking out of fire, light radiations fall upon the photocell. This completes the electric circuit through an electric bell or a siren which starts operating as a warning signal.

Photocells are used in the reproduction of sound in motion pictures and in the television camera for scanning and telecasting scenes. They are used in industries for detecting minor flaws or holes in metal sheets.

$$p = mv = 0.12 \text{ kg} \times 20 \text{ m s}^{-1} = 2.40 \text{ kg m s}^{-1}$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J s}}{2.40 \text{ kg m s}^{-1}} = 2.76 \times 10^{-34} \text{ m}$$

LOUIS VICTOR DE BROGLIE (1892 – 1987)

Louis Victor de Broglie (1892 – 1987) French physicist who put forth revolutionary idea of wave nature of matter. This idea was developed by Erwin Schródinger into a full-fledged theory of quantum mechanics commonly known as wave mechanics. In 1929, he was awarded the Nobel Prize in Physics for his discovery of the wave nature of electrons.

This wavelength is so small that it is beyond any measurement. This is the reason why macroscopic objects in our daily life do not show wave-like properties. On the other hand, in the sub-atomic domain, the wave character of particles is significant and measurable.

Consider an electron (mass m, charge e) accelerated from rest through a potential V. The kinetic energy K of the electron equals the work done (eV) on it by the electric field:

$$K = e\,V \tag{11.8}$$

Now, $K = \frac{1}{2} m v^2 = \frac{p^2}{2m}$, so that

$$p = \sqrt{2\,m\,K} = \sqrt{2\,m\,eV} \tag{11.9}$$

The de Broglie wavelength λ of the electron is then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\,mK}} = \frac{h}{\sqrt{2\,meV}} \tag{11.10}$$

Substituting the numerical values of h, m, e, we get

$$\lambda = \frac{1.227}{\sqrt{V}}\ \text{nm} \tag{11.11}$$

where V is the magnitude of accelerating potential in volts. For a 120 V accelerating potential, Eq. (11.11) gives λ = 0.112 nm. This wavelength is of the same order as the spacing between the atomic planes in crystals. This suggests that matter waves associated with an electron could be verified by crystal diffraction experiments analogous to X-ray diffraction. We describe the experimental verification of the de Broglie hypothesis in the next section. In 1929, de Broglie was awarded the Nobel Prize in Physics for his discovery of the wave nature of electrons.

The matter–wave picture elegantly incorporated the Heisenberg's *uncertainty principle*. According to the principle, it is not possible to measure *both* the position and momentum of an electron (or any other particle) *at the same time* exactly. There is always some uncertainty (Δx) in the specification of position and some uncertainty (Δp) in the specification of momentum. The product of Δx and Δp is of the order of $\hbar$* (with $\hbar = h/2\pi$), i.e.,

$$\Delta x\,\Delta p \approx \hbar \tag{11.12}$$

Equation (11.12) allows the possibility that Δx is zero; but then Δp must be infinite in order that the product is non-zero. Similarly, if Δp is zero, Δx must be infinite. Ordinarily, both Δx and Δp are non-zero such that their product is of the order of $\hbar$.

Now, if an electron has a definite momentum p, (i.e.$\Delta p = 0$), by the de Broglie relation, it has a definite wavelength λ. A wave of definite (single)

* A more rigorous treatment gives $\Delta x\,\Delta p \geq \hbar/2$.

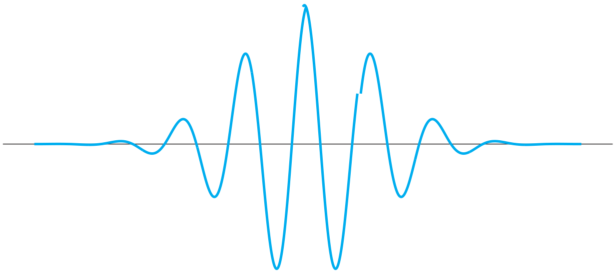
wavelength extends all over space. By Born's probability interpretation this means that the electron is not localised in any finite region of space. That is, its position uncertainty is infinite ($\Delta x \to \infty$), which is consistent with the uncertainty principle.

In general, the matter wave associated with the electron is not extended all over space. It is a wave packet extending over some finite region of space. In that case Δx is not infinite but has some finite value depending on the extension of the wave packet. Also, you must appreciate that a wave packet of finite extension does not have a single wavelength. It is built up of wavelengths spread around some central wavelength.

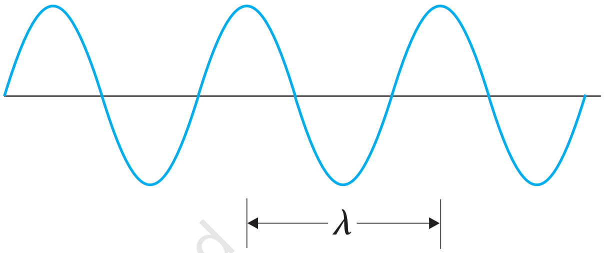
By de Broglie's relation, then, the momentum of the electron will also have a spread – an uncertainty Δp. This is as expected from the uncertainty principle. It can be shown that the wave packet description together with de Broglie relation and Born's probability interpretation reproduce the Heisenberg's uncertainty principle exactly.

In Chapter 12, the de Broglie relation will be seen to justify Bohr's postulate on quantisation of angular momentum of electron in an atom.

Figure 11.6 shows a schematic diagram of (a) a localised wave packet, and (b) an extended wave with fixed wavelength.



(a)



(b)

FIGURE 11.6 (a) The wave packet description of an electron. The wave packet corresponds to a spread of wavelength around some central wavelength (and hence by de Broglie relation, a spread in momentum). Consequently, it is associated with an uncertainty in position (Δx) and an uncertainty in momentum (Δp). (b) The matter wave corresponding to a definite momentum of an electron extends all over space. In this case, $\Delta p = 0$ and $\Delta x \to \infty$.

Example 11.4 What is the de Broglie wavelength associated with (a) an electron moving with a speed of 5.4×10^6 m/s, and (b) a ball of mass 150 g travelling at 30.0 m/s?

Solution

(a) For the electron:

Mass $m = 9.11\times10^{-31}$ kg, speed $v = 5.4\times10^6$ m/s. Then, momentum

$p = m\,v = 9.11\times10^{-31}$ (kg) $\times$ 5.4 $\times$ 10^6 (m/s)

$p = 4.92 \times 10^{-24}$ kg m/s

de Broglie wavelength, $\lambda = h/p$

$$= \frac{6.63\times10^{-34}\ \text{J s}}{4.92\times10^{-24}\ \text{kg m/s}}$$

$\lambda = 0.135$ nm

(b) For the ball:

Mass $m' = 0.150$ kg, speed $v' = 30.0$ m/s.

Then momentum $p' = m'\,v' = 0.150$ (kg) $\times$ 30.0 (m/s)

$p' = 4.50$ kg m/s

de Broglie wavelength $\lambda' = h/p'$.

EXAMPLE 11.4

EXAMPLE 11.4

$$= \frac{6.63 \times 10^{-34} \text{ J s}}{4.50 \times \text{ kg m/s}}$$

$\lambda' = 1.47 \times 10^{-34}$ m

The de Broglie wavelength of electron is comparable with X-ray wavelengths. However, for the ball it is about 10^{-19} times the size of the proton, quite beyond experimental measurement.

EXAMPLE 11.5

Example 11.5 An electron, an α-particle, and a proton have the same kinetic energy. Which of these particles has the shortest de Broglie wavelength?

Solution

For a particle, de Broglie wavelength, $\lambda = h/p$

Kinetic energy, $K = p^2/2m$

Then, $\lambda = h/\sqrt{2mK}$

For the same kinetic energy K, the de Broglie wavelength associated with the particle is inversely proportional to the square root of their masses. A proton $\left({}^{1}_{1}\text{H}\right)$ is 1836 times massive than an electron and an α-particle $\left({}^{4}_{2}\text{He}\right)$ four times that of a proton.

Hence, α – particle has the shortest de Broglie wavelength.

PROBABILITY INTERPRETATION TO MATTER WAVES

It is worth pausing here to reflect on just what a matter wave associated with a particle, say, an electron, means. Actually, a truly satisfactory physical understanding of the dual nature of matter and radiation has not emerged so far. The great founders of quantum mechanics (Niels Bohr, Albert Einstein, and many others) struggled with this and related concepts for long. Still the deep physical interpretation of quantum mechanics continues to be an area of active research. Despite this, the concept of matter wave has been mathematically introduced in modern quantum mechanics with great success. An important milestone in this connection was when Max Born (1882-1970) suggested a probability interpretation to the matter wave amplitude. According to this, the intensity (square of the amplitude) of the matter wave at a point determines the probability density of the particle at that point. Probability density means probability per unit volume. Thus, if A is the amplitude of the wave at a point, $|A|^2 \Delta V$ is the probability of the particle being found in a small volume ΔV around that point. Thus, if the intensity of matter wave is large in a certain region, there is a greater probability of the particle being found there than where the intensity is small.

EXAMPLE 11.6

Example 11.6 A particle is moving three times as fast as an electron. The ratio of the de Broglie wavelength of the particle to that of the electron is 1.813×10^{-4}. Calculate the particle's mass and identify the particle.

Solution

de Broglie wavelength of a moving particle, having mass m and velocity v:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Mass, $m = h/\lambda v$

For an electron, mass $m_e = h/\lambda_e v_e$

Now, we have $v/v_e = 3$ and

$\lambda/\lambda_e = 1.813 \times 10^{-4}$

Then, mass of the particle, $m = m_e \left(\frac{\lambda_e}{\lambda}\right)\left(\frac{v_e}{v}\right)$

$m = (9.11\times10^{-31}$ kg$) \times (1/3) \times (1/1.813 \times 10^{-4})$

$m = 1.675 \times 10^{-27}$ kg.

Thus, the particle, with this mass could be a proton or a neutron.

EXAMPLE 11.6

Example 11.7 What is the de Broglie wavelength associated with an electron, accelerated through a potential differnece of 100 volts?

Solution Accelerating potential $V = 100$ V. The de Broglie wavelength λ is

$$\lambda = h/p = \frac{1.227}{\sqrt{V}} \text{ nm}$$

$$\lambda = \frac{1.227}{\sqrt{100}} \text{ nm} = 0.123 \text{ nm}$$

The de Broglie wavelength associated with an electron in this case is of the order of X-ray wavelengths.

EXAMPLE 11.7

11.9 Davisson and Germer Experiment

The wave nature of electrons was first experimentally verified by C.J. Davisson and L.H. Germer in 1927 and independently by G.P. Thomson, in 1928, who observed diffraction effects with beams of electrons scattered by crystals. Davisson and Thomson shared the Nobel Prize in 1937 for their experimental discovery of diffraction of electrons by crystals.

The experimental arrangement used by Davisson and Germer is schematically shown in Fig. 11.7. It consists of an electron gun which comprises of a tungsten filament F, coated with barium oxide and heated by a low voltage power supply (L.T. or battery). Electrons emitted by the filament are accelerated to a desired velocity

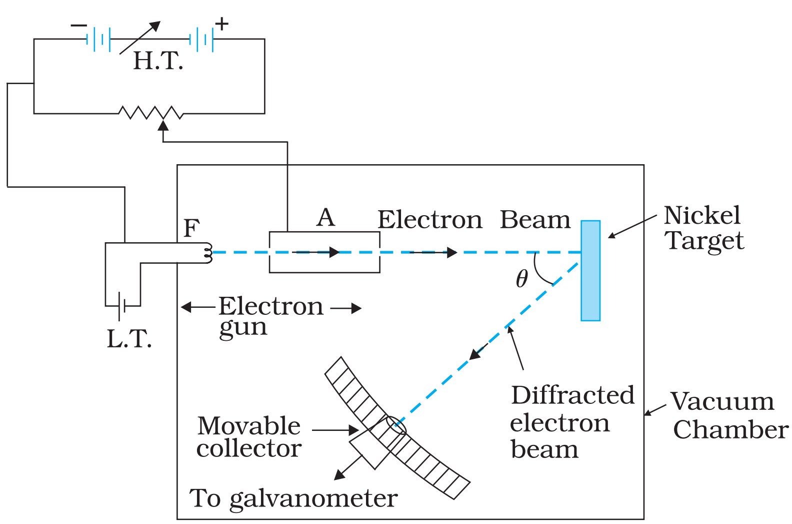


FIGURE 11.7 Davisson-Germer electron diffraction arrangement.

by applying suitable potential/voltage from a high voltage power supply (H.T. or battery). They are made to pass through a cylinder with fine holes along its axis, producing a fine collimated beam. The beam is made to fall on the surface of a nickel crystal. The electrons are scattered in all directions by the atoms of the crystal. The intensity of the electron beam, scattered in a given direction, is measured by the electron detector (collector). The detector can be moved on a circular scale and is connected to a sensitive galvanometer, which records the current. The deflection of the galvanometer is proportional to the intensity of the electron beam entering the collector. The apparatus is enclosed in an evacuated chamber. By moving the detector on the circular scale at different positions, the intensity of the scattered electron beam is measured for different values of angle of scattering θ which is the angle between the incident and the scattered electron beams. The variation of the intensity (I) of the scattered electrons with the angle of scattering θ is obtained for different accelerating voltages.

The experiment was performed by varying the accelarating voltage from 44 V to 68 V. It was noticed that a strong peak appeared in the intensity (I) of the scattered electron for an accelarating voltage of 54V at a scattering angle $\theta = 50°$

The appearance of the peak in a particular direction is due to the constructive interference of electrons scattered from different layers of the regularly spaced atoms of the crystals. From the electron diffraction measurements, the wavelength of matter waves was found to be 0.165 nm.

The de Broglie wavelength λ associated with electrons, using Eq. (11.11), for $V = 54$ V is given by

$$\lambda = h/p = \frac{1\ 227}{\sqrt{V}} \text{ nm}$$

$$\lambda = \frac{1\ 227}{\sqrt{54}} \text{ nm} = 0.167 \text{ nm}$$

Thus, there is an excellent agreement between the theoretical value and the experimentally obtained value of de Broglie wavelength. Davisson-Germer experiment thus strikingly confirms the wave nature of electrons and the de Broglie relation. More recently, in 1989, the wave nature of a beam of electrons was experimentally demonstrated in a double-slit experiment, similar to that used for the wave nature of light. Also, in an experiment in 1994, interference fringes were obtained with the beams of iodine molecules, which are about a million times more massive than electrons.

The de Broglie hypothesis has been basic to the development of modern quantum mechanics. It has also led to the field of electron optics. The wave properties of electrons have been utilised in the design of electron microscope which is a great improvement, with higher resolution, over the optical microscope.

Development of electron microscope
http://www.nobelprize.org/nobel_prizes/physics/laureates/1986/presentation-speech.html

PHYSICS