If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right) dx = A(x)\tan^{-1}(\sqrt{x}) + B(x) + C$, wher e C is a constant of integration, then the ordered pair $(A(x), B(x))$ can be :

[Main Sep. 03, 2020 (II)]

(a) $(x+1, -\sqrt{x})$

(b) $(x+1, \sqrt{x})$

(c) $(x-1, -\sqrt{x})$

(d) $(x-1, \sqrt{x})$

$$I = \int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right) dx = \int \tan^{-1}\sqrt{x} \cdot 1dx$$

$$= x\tan^{-1}\sqrt{x} - \int \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} \cdot xdx + C$$

$$= x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{t \cdot 2tdt}{1+t^2} + C \text{ (Put } x = t^2 \Rightarrow dx = 2tdt\text{)}$$

$$= x\tan^{-1}\sqrt{x} - \int \frac{t^2}{1+t^2} dt + C$$

$$= x\tan^{-1}\sqrt{x} - t + \tan^{-1} t + C$$

$$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$$

$$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$$

$$\Rightarrow A(x) = x + 1 \Rightarrow B(x) = -\sqrt{x}$$

The correct option is (a)