

$$\int \frac{4x+1}{x^2+3x+2} dx.$$

$$\begin{aligned} \text{Answer: } I &= \int \frac{4x+1}{x^2+3x+2} dx \\ &= \int \frac{2(2x+3)-5}{x^2+3x+2} dx \\ &= 2 \int \frac{2x+3}{x^2+3x+2} dx - 5 \int \frac{1}{x^2+3x+2} dx \\ &= 2 \log |x^2+3x+2| - 5 \int \frac{1}{x^2+3x+(9/4)-(9/4)+2} dx \\ &= 2 \log |x^2+3x+2| - 5 \int \frac{1}{(x+3/2)^2 - (1/2)^2} dx \\ &= 2 \log |x^2+3x+2| - 5 \frac{1}{2(1/2)} \log \left| \frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}} \right| + C \\ &= 2 \log |x^2+3x+2| - 5 \log \left| \frac{x+1}{x+2} \right| + C \end{aligned}$$