

$$\int \sqrt{\frac{1+x}{x}} dx$$

Answer:  $\int \sqrt{\frac{1+x}{x}} dx.$

$$\begin{aligned} &= \int \sqrt{\frac{1+x}{x}} \sqrt{\frac{1+x}{1+x}} dx = \int \frac{1+x}{\sqrt{x(1+x)}} dx \\ &= \int \frac{1+x}{\sqrt{x^2+x}} dx \\ &= \frac{1}{2} \int \frac{2x+1}{\sqrt{x^2+x}} dx + \frac{1}{2} \int \frac{1}{\sqrt{x^2+x}} dx \\ &= \frac{1}{2} \int \frac{1}{\sqrt{t}} dt + \frac{1}{2} \int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx, \text{ where } t = x^2 + x \\ &= \sqrt{t} + \frac{1}{2} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x} \right| + C \\ &= \sqrt{x^2 + x} + \frac{1}{2} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x} \right| + C \end{aligned}$$