Q3) $\displaystyle\int \frac{1}{x^4+5x^2+1}\,dx$

$$I = \frac{1}{2}\int \frac{2}{x^4+5x^2+1} = \frac{1}{2}\int \frac{1+x^2}{x^4+5x^2+1} - \frac{1}{2}\int \frac{1-x^2}{x^4+5x^2+1}$$

$$I = \frac{1}{2}\left[\int \frac{\left(1+\frac{1}{x^2}\right)dx}{x^2+\frac{1}{x^2}+5} - \int \frac{\left(1-\frac{1}{x^2}\right)dx}{x^2+\frac{1}{x^2}+5}\right]$$

$t = x - \frac{1}{x}$

$dt = 1 - \frac{1}{x^2}$

$u = x + \frac{1}{x}$

$du = 1 - \frac{1}{x^2}$

$$I = \frac{1}{2}\int \frac{dt}{t^2+7} - \frac{1}{2}\int \frac{du}{u^2+3}$$

$$I = \frac{1}{2}\int \frac{dt}{t^2+(\sqrt{7})^2} - \frac{1}{2}\int \frac{du}{u^2+(\sqrt{3})^2}$$

$$I = \frac{1}{2\sqrt{7}}\left(\tan^{-1}\left(\frac{t}{\sqrt{7}}\right) - \frac{1}{2}\times\frac{1}{\sqrt{3}}\left(\tan^{-1}\left(\frac{u}{\sqrt{3}}\right)\right)\right) + c$$

$$I = \frac{1}{2\sqrt{7}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{7}}\right) - \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{1}{x}}{\sqrt{3}}\right) + c$$