Q1) $\int \frac{dx}{\cos^6 x + \sin^6 x}$ is equal to :-

$$\int \frac{dx}{\cos^6 x + \sin^6 x} = \int \frac{\sec^6 x\, dx}{1 + \tan^6 x}$$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$\Rightarrow \int \frac{(1+t^2)^2\, dt}{1+t^6} = \int \frac{(1+t^2)\, dt}{t^4 - t^2 + 1} = \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t^2 - 1 + \frac{1}{t^2}\right)}$$

Let $t - \frac{1}{t} = u \Rightarrow \left(1 + \frac{1}{t^2}\right) dt = du$

$$\int \frac{du}{u^2+1} = \tan^{-1} u + c = \tan^{-1}\left(t - \frac{1}{t}\right) + c$$

$$= \tan^{-1}(\tan x - \sec x) + c$$