4. Let P be the point on the parabola $y^2 = 4x$, which is at the shortest distance from the centre S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then,

(a) $SP = 2\sqrt{5}$ **(2016 Adv.)**

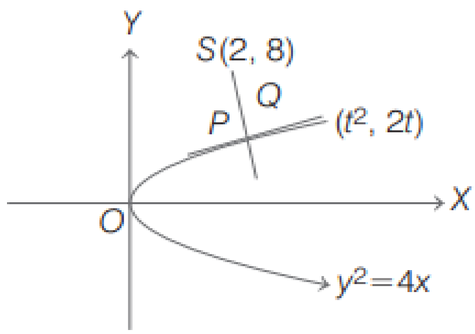
(b) $SQ : QP = (\sqrt{5} + 1) : 2$

(c) the x-intercept of the normal to the parabola at P is 6

(d) the slope of the tangent to the circle at Q is $\frac{1}{2}$

Solution: -

4. Tangent to $y^2 = 4x$ at $(t^2, 2t)$ is



$$y(2t) = 2(x + t^2)$$

$\Rightarrow$ $yt = x + t^2$ ...(i)

Equation of normal at $P(t^2, 2t)$ is

$$y + tx = 2t + t^3$$

Since, normal at P passes through centre of circle $S(2, 8)$.

$\therefore$ $8 + 2t = 2t + t^3$

$\Rightarrow$ $t = 2$, i.e. $P(4, 4)$

[since, shortest distance between two curves lie along their common normal and the common normal will pass through the centre of circle]

$\therefore$ $SP = \sqrt{(4-2)^2 + (4-8)^2} = 2\sqrt{5}$

$\therefore$ Option (a) is correct.

Also, $SQ = 2$

$\therefore$ $PQ = SP - SQ = 2\sqrt{5} - 2$

Thus, $\dfrac{SQ}{QP} = \dfrac{1}{\sqrt{5} - 1} = \dfrac{\sqrt{5} + 1}{4}$

$\therefore$ Option (b) is wrong.

Now, x-intercept of normal is $x = 2 + 2^2 = 6$

$\therefore$ Option (c) is correct.

Slope of tangent $= \dfrac{1}{t} = \dfrac{1}{2}$

$\therefore$ Option (d) is correct.