. Let f: R -> R be such that for all x ∈ R, $(2^{1+x} + 2^{1-x})$, f(x) and $(3^x + 3^{-x})$ are in A.P., then the minimum value of f(x) is :

(a) 0 (b) 4 (c) 3 (d) 2

Answer: (c)

Solution:

Given: $(2^{1+x} + 2^{1-x})$, f(x) and $(3^x + 3^{-x})$ are in A.P.

Therefore,

$$f(x) = \frac{3^x + 3^{-x} + 2^{1+x} + 2^{1-x}}{2} = \frac{(3^x + 3^{-x})}{2} + \frac{2^{1+x} + 2^{1-x}}{2}$$

Applying, A.M. ≥ G.M. inequality,

$$\frac{(3^x + 3^{-x})}{2} \geq \sqrt{3^x . 3^{-x}}$$

$$\Rightarrow \frac{(3^x + 3^{-x})}{2} \geq 1 \qquad \text{... (1)}$$

By A.M. ≥ G.M. inequality,

$$\frac{2^{1+x} + 2^{1-x}}{2} \geq \sqrt{2^{1+x} . 2^{1-x}}$$

$$\Rightarrow \frac{2^{1+x} + 2^{1-x}}{2} \geq 2 \quad \text{... (2)}$$

Add (1) and (2)

$f(x) \geq 1 + 2 = 3$

Thus, minimum value of f(x) is 3. Answer!