Prove (a + b + c) (ab + bc + ca) > 9abc.

Solution:

$\dfrac{a+b+c}{3}$

> (abc)$^{1/3}$ and

$\dfrac{ab+bc+ca}{3}$

>

$(ab.\,bc.\,ca)^{\frac{1}{3}}$

.

[Using AM > GM]

Multiplying above results, we get

$\dfrac{a+b+c}{3} \times \dfrac{ab+bc+ca}{3}$

> (abc)$^{1/3}$ (ab.bc.ca)$^{1/3}$

= (a^{3}b^{3}c^3)$^{1/3}$

= abc

=> (a + b + c)(ab + bc + ca) > 9abc.

Hence Proved.