S_n denotes the sum of cubes of first n natural numbers and t_n denotes the sum of first n natural numbers then find the value of $\sum_{r=1}^{n} \frac{S_r}{t_r}$

SOLUTION :

S_n is sum of cubes of first n natural numbers.

$\therefore S_n = \frac{(n)^2 (n+1)^2}{4}$

t_n is sum of first n natural numbers

$\therefore t_n = \frac{(n)(n+1)}{2}$

now, $\frac{S_n}{t_n} = \frac{(n)(n+1)}{2} = \frac{n^2+n}{2}$

$\therefore \sum_{r=1}^{r=n} \frac{S_r}{t_r} = \sum_{r=1}^{n} \frac{r^2+r}{2}$

$= \frac{(n)(n+1)(2n+1)}{12} + \frac{(n)(n+1)}{4}$

$= \frac{(n)(n+1)}{4} \left(\frac{2n+1}{3} + 1 \right)$

$= \frac{(n)(n+1)}{4} \cdot \frac{(2n+4)}{3}$

$= \frac{(n)(n+1)(n+2)}{6}$