Find n such that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the AM between a and b

SOLUTION :

$$\frac{a+b}{2} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$$

$$a^{n+1}+ba^n+b^{n+1}+ab^n = 2a^{n+1}+2b^{n+1}$$

$$ba^n+ab^n = a^{n+1}+b^{n+1}$$

$$ab^n-b^{n+1} = a^{n+1}-ba^n$$

$$b^n(a-b) = a^n(a-b)$$

$$\Rightarrow \quad a^n = b^n \qquad (\because a \neq b)$$

$$\Rightarrow \quad \left(\frac{a}{b}\right)^n = 1$$

$$\Rightarrow \quad n = 0$$