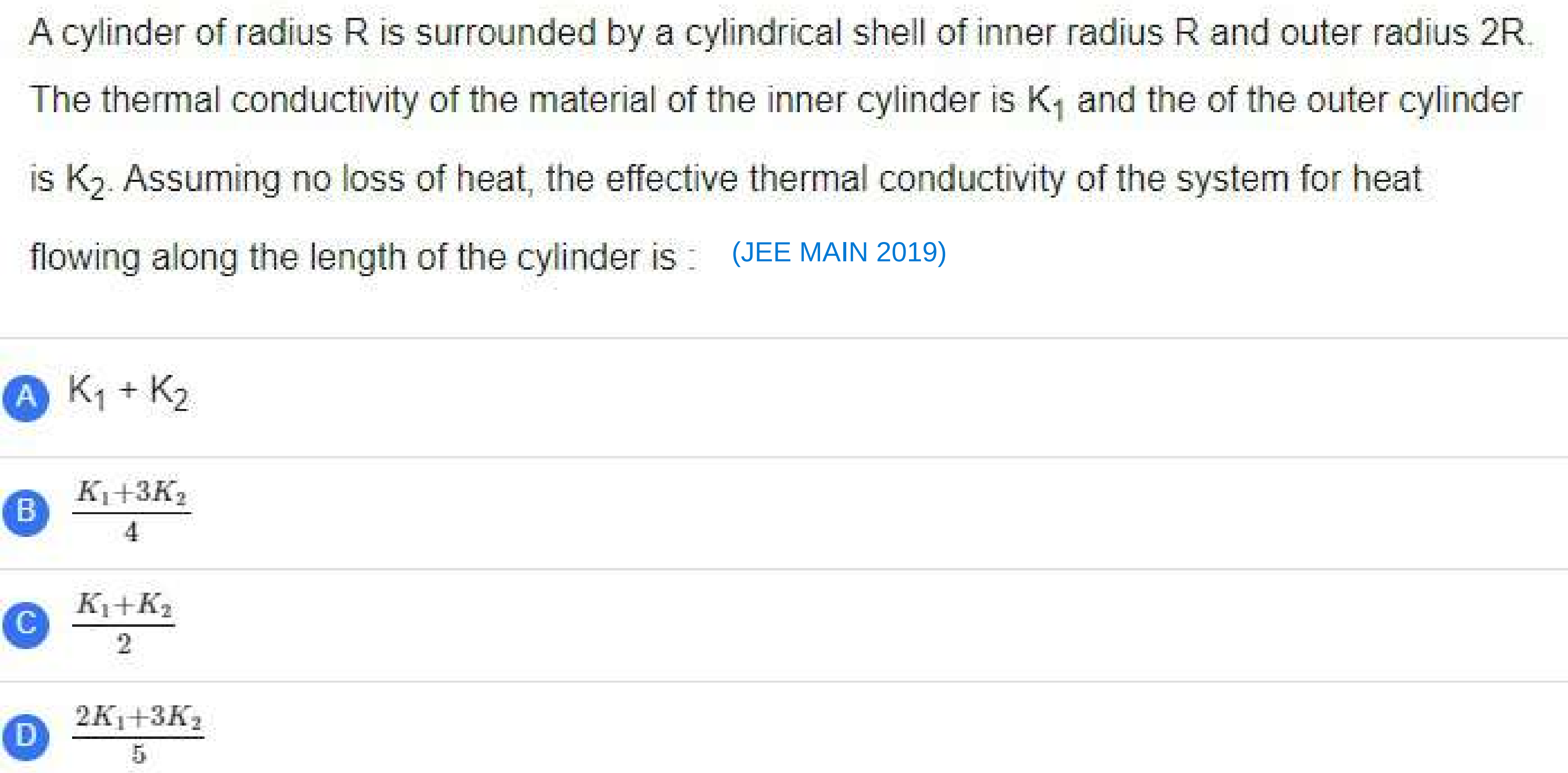

A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and the of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is : (JEE MAIN 2019)

(A) $K_1 + K_2$

(B) $\frac{K_1+3K_2}{4}$

(C) $\frac{K_1+K_2}{2}$

(D) $\frac{2K_1+3K_2}{5}$

Here, the cylinders are in parallel combination (as temperature difference will be same along their length)

$$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \cancel{\frac{l_{eff}}{K_{eff}. A_{eff}}} \cancel{=} \cancel{\frac{l_1}{K_1 A_1}} \cancel{+} \cancel{\frac{l_2}{K_2 A_2}} \quad \frac{K_{eff} A_{eff}}{l_{eff}} = \frac{K_1 A_1}{l_1} + \frac{K_2 A_2}{l_2}$$

Here, $l_1 = l_2 = l$; $A_1 = \pi(R)^2$; $A_{eff} = \pi(2R)^2$

$$= 4\pi R^2$$

$$A_2 = \pi(2R)^2 - \pi(R)^2$$

$$= 3\pi R^2$$

$$\therefore \cancel{\frac{4}{K_{eff}}} \quad \frac{K_{eff}(4\pi R^2)}{l} = \frac{K_1(\pi R^2)}{l} + \frac{K_2(3\pi R^2)}{l}$$

$$\Rightarrow \boxed{K_{eff} = \frac{K_1 + 3K_2}{4}}$$